Ionosphere-Distance Calculations

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A. Great circle distance:

The total great circle arc angle Φ and distance d between two stations are found from the station coordinates (latitude and longitude).

$$\cos(\Phi) = \sin A \cdot \sin C + \cos A \cdot \cos C \cdot \cos \Delta L \qquad (1)$$

where (all angles use the same units, all degrees or all radians)
Φ = Angle of the great circle arc from transmitter a and receiver c
A = latitude of station a (+ for northern hemisphere)
C = latitude of station c (+ for northern hemisphere)
ΔL = angular difference in longitude between stations a and c (magnitude > 0)

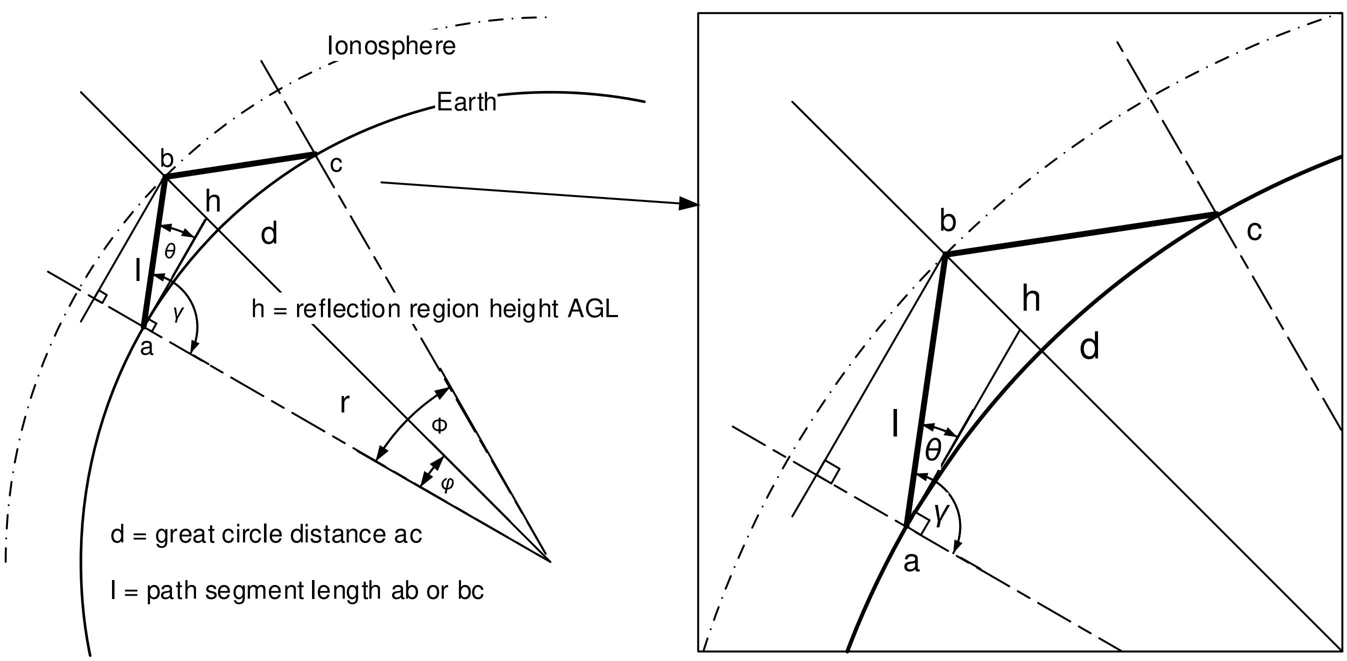
$$\Phi = arc\cos(\sin A \cdot \sin C + \cos A \cdot \cos C \cdot \cos \Delta L) \qquad (2)$$

$$d = r \cdot \Phi = 111.2 \cdot \Phi^{\circ} \text{ km} \qquad (3.a)$$

$$d = r \cdot \Phi = 6370 \cdot \Phi^{rad} \text{ km} \qquad (3.b)$$

For additional details on position, distance and bearing calculations, see [PDBC].

B. Radio path length:



A hop is defined as the radio path from the ground up to the reflection (refraction) region and back down to the ground. For a 1-hop circuit a-b-c, the total radio path length is $2 \cdot l$ and for an m-hop circuit the total path length is $2 \cdot m \cdot l$.

$$\varphi = \frac{\Phi}{2 \cdot m} \tag{4}$$

From Law of Cosines, the radio path segment length l is given by

$$l^2 = 2 \cdot r \cdot (r+h) \cdot (1-\cos\varphi) + h^2$$

$$l = \sqrt{2 \cdot r \cdot (r+h) \cdot (1-\cos\varphi) + h^2} \tag{5}$$

where

φ = Angle of the great circle arc from transmitter a (or receiver c) and point on Earth below reflection (refraction) region b

l = path segment length from transmitter a (or receiver c) to reflection (refraction) region b (km)

r = Earth radius (6370 km)

h = height of reflection region above ground level (km)

C. Elevation angle:

By inspection

$$l \cdot \sin(\pi - \gamma) = l \cdot \sin\gamma = (r+h) \cdot \sin\phi \tag{6}$$

and

$$\sin\gamma = \frac{(r+h) \cdot \sin\varphi}{l} \tag{7}$$

where

γ = angle between Earth radial to transmitter a (or receiver c) and radio path l. Note: γ > $\frac{\pi}{2}$ rad.

Therefore,

$$\sin\gamma = \frac{(r+h) \cdot \sin\varphi}{\sqrt{2 \cdot r \cdot (r+h) \cdot (1-\cos\varphi) + h^2}} \tag{8}$$

and

$$\theta = arc\sin\gamma - \frac{\pi}{2} \tag{9}$$

ϑ = elevation angle (Note: ϑ must be ≥ 0)

References:

[PDBC] Reeve, W., Position, Distance and Bearing Calculations, 2014, http://www.reeve.com/Documents/Articles%20Papers/Reeve_PosDistBrngCalcs.pdf

[SFD] Reeve, W., Sudden Frequency Deviations Due to Solar Flares, 2014, http://www.reeve.com/Documents/Articles%20Papers/Reeve_SudFreqDev.pdf

Document Information

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