

Position, Distance and Bearing Calculations

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1. Introduction

Two frequent calculations required in radio propagation work are distance and bearing between two radio terminals. Distance and bearing may be taken from topographical maps for quick analyses but great circle calculations are needed for more accurate work and are described here. Online resources also may be used to calculate great circle distances; for example, http://www.onlineconversion.com/map_greatcircle_distance.htm

In this paper west longitudes and north latitudes are positive numbers (0° to $+180^\circ$ and 0° to $+90^\circ$, respectively). For our purposes, bearing and azimuth are taken to be the same thing: the horizontal arc expressed as the clockwise angle subtended from some reference direction (usually true north or magnetic north) to the point in question.

2. Position

The first step in finding distance and bearing is determining the position (geographical coordinates) of the radio terminals. The position may be taken from a topographical map. However, this can be inaccurate if the topographical maps are old and for remote areas or if the maps are small scale such as 1:250 000 (1 in = 3.95 mi) or 1:63 360 scale (1 in = 1 mi). Both scales are too small to accurately determine the position of a point. Instead, maps with 1:25 000 scale or higher should be used. The most accurate way of determining the position, in almost all situations, is to use a GPS (Global Positioning System) receiver.

Make all calculations to four or five decimal places. If the position is in degrees, minutes and seconds dd+mm+ss, the reading should be converted to decimal degrees dd.dddd with

$$dd.dddd = dd + \frac{mm}{60} + \frac{ss}{3600} \quad (1)$$

Procedures:

- 1) Divide the number of seconds by 60 to obtain decimal minutes (.mm = ss/60)
- 2) Add the decimal minutes to the minutes (mm.mm = mm + ss/60);
- 3) Divide the sum by 60 to obtain decimal degrees (.dd = mm.mm/60 = mm/60 + ss/3600);
- 4) Add the decimal degrees to the degrees [dd.dd = dd + mm.mm/60 = dd + (mm + ss/60)/60 = dd + mm/60 + ss/3600].

Example: Convert $66^\circ 53' 43.2''$ north latitude to decimal degrees.

$$43.2/3600 = 0.0120$$

$$53/60 = 0.8833$$

$$66 + 0.8833 + 0.0120 = 66.8953^\circ$$

If the geographical coordinates are to be used in a spreadsheet program, the degrees must be converted to radians from (some spreadsheet programs have built-in functions for this conversion)

$$a = \frac{\pi}{180} \cdot b \tag{2}$$

where

a = latitude or longitude in radians

b = latitude or longitude in degrees

If it is necessary to convert from radians to degrees use

$$b = \frac{180}{\pi} \cdot a \tag{3}$$

Example: Convert the latitude in degrees found in the previous example to radians:

$$\frac{\pi}{180} \cdot 66.8593^\circ = 11.669 \text{ rad}$$

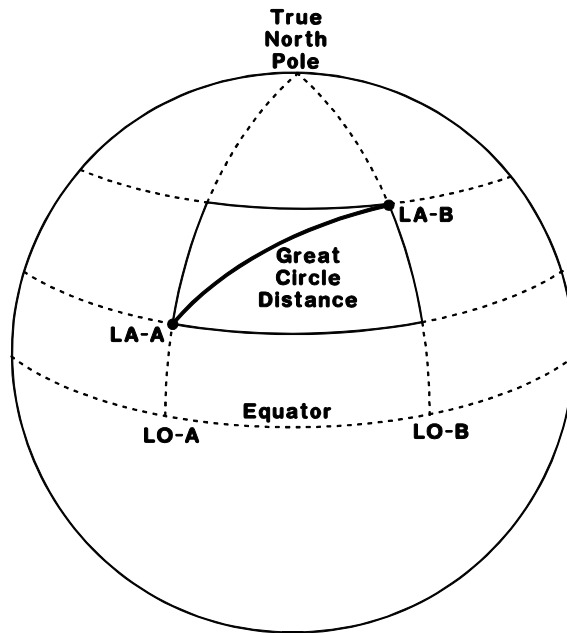


Figure 1 ~ Nomenclature for Great Circle Path Computations

The great circle formulas for distance and bearings in the northern hemisphere are given by (figure 1):

$$\Phi_C = LO_A - LO_B \text{ degrees} \tag{4}$$

$$\Theta_{A \rightarrow B} = \arctan \left[\frac{\sin(\Phi_C)}{\cos(LA_A) \cdot \tan(LA_B) - \sin(LA_A) \cdot \cos(\Phi_C)} \right] \text{ degrees true} \quad (5)$$

$$\Theta_{B \rightarrow A} = \arctan \left[\frac{\sin(\Phi_C)}{\cos(LA_B) \cdot \tan(LA_A) - \sin(LA_B) \cdot \cos(-\Phi_C)} \right] \text{ degrees true} \quad (6)$$

$$L_{A \rightarrow B} = L_{B \rightarrow A} = 111.19 \cdot \arccos \left[\sin(LA_A) \cdot \sin(LA_B) + \cos(LA_A) \cdot \cos(LA_B) \cdot \cos(\Phi_C) \right] \text{ km} \quad (7)$$

$$L_{A \rightarrow B} = L_{B \rightarrow A} = 69.09 \cdot \arccos \left[\sin(LA_A) \cdot \sin(LA_B) + \cos(LA_A) \cdot \cos(LA_B) \cdot \cos(\Phi_C) \right] \text{ mi}$$

where

LO_A = longitude of point A

LO_B = longitude of point B

Φ_C = difference in longitude between points A and B

LA_A = latitude of point A

LA_B = latitude of point B

$\theta_{A \rightarrow B}$ = true bearing from point A to B

$\theta_{B \rightarrow A}$ = true bearing from point B to A

$L_{A \rightarrow B} = L_{B \rightarrow A}$ = great circle distance between points A and B, kilometers or statute miles.

The above expressions assume the arguments in all trigonometric functions and results are in degrees. If the inverse cosine function returns a value in radians for the distance calculation, substitute the following constants:

- For a final result in kilometers, substitute 111.19 with 6370
- For a final result in statute miles, substitute 69.09 with 3,959

The bearings require modification according to table 1. This judgement table removes any quadrant or 180° ambiguity and ensures the results are positive bearings between 0° and 360°.

There is one special case not covered in the judgement table. If the longitudes of both sites are exactly the same, the expression does not resolve which bearing is south and which is north. Therefore, it is necessary to inspect the latitudes and assign the direction according to Table 2.

Maps are based on a horizontal datum, such as the North American Datum of 1927 (NAD-27), but some analyses require other datums, such as WGS-72, WGS-84 and NAD-83. Care must be taken in the use of maps and GPS receivers and different datums. Errors up to 100 m may arise between NAD-27 and NAD-83. The conversion between NAD-27 and NAD-83 (which is essentially the same as WGS-84 and WGS-72) is quite tedious. Conversion information may be obtained from the US Geological Survey (USGS):

<http://www.ngs.noaa.gov/TOOLS/Nadcon/Nadcon.shtml>.

When a GPS receiver is used to determine the coordinates, it should be set for the desired datum when the measurements are made. Most GPS receivers can save the measured coordinates as a waypoint. The displayed

coordinates will reflect the datum set in the receiver. Conversion between datums can be done simply by changing the datum setting, which will automatically update the displayed waypoint.

Table 1 ~ Bearing Judgement Table. $B_{A \rightarrow B}$ and $B_{B \rightarrow A}$ are the resolved bearings between 0° and 360°
 Examples: If the sign of $\sin(\Phi_C)$ is + and the sign of $\theta_{A \rightarrow B}$ is +, then bearing $B_{A \rightarrow B} = \theta_{A \rightarrow B}$; if the sign of $\sin(\Phi_C)$ is + and the sign of $\theta_{A \rightarrow B}$ is -, then bearing $B_{A \rightarrow B} = \theta_{A \rightarrow B} + 180^\circ$

sign $\sin(\Phi_C)$ sign $\sin(-\Phi_C)$	sign $\theta_{A \rightarrow B}$ sign $\theta_{B \rightarrow A}$	substitute $B_{A \leftrightarrow B}$ $B_{B \rightarrow A}$
+	+	$\theta_{A \rightarrow B}$ $\theta_{B \rightarrow A}$
+	-	$\theta_{A \rightarrow B} + 180$ $\theta_{B \rightarrow A} + 180$
-	+	$\theta_{A \rightarrow B} + 180$ $\theta_{B \rightarrow A} + 180$
-	-	$\theta_{A \rightarrow B} + 360$ $\theta_{B \rightarrow A} + 360$

Table 2 ~ North-South Judgement Table

Latitudes	$B_{A \rightarrow B}$	$B_{B \rightarrow A}$
$LA_A < LA_B$	180°	0°
$LA_A > LA_B$	0°	180°

When using a compass in the field to point antennas, it is necessary to convert the true north bearings to magnetic north bearings. To do this, the magnetic declination is taken from a USGS map, aeronautical chart or online source such as <http://www.ngdc.noaa.gov/geomag-web/>. If the declination is easterly, it must be subtracted from the true north bearing taken from a map. For example, if the bearing from Site A to Site B is found to be 293° from true north and the declination is 22° east, the magnetic bearing is $(293^\circ - 22^\circ =) 271^\circ$. This procedure can be reversed to find magnetic bearing if true bearing is known. For example, if the true bearing is 10° , the magnetic bearing would be $(10^\circ - 22^\circ + 360^\circ =) 348^\circ$. Similarly, a measured bearing of 95° magnetic would be equivalent to $(95^\circ + 22^\circ =) 117^\circ$ true.

If more than a few sites are being evaluated, it can be more productive to use a programmable calculator or spreadsheet program to find distance and bearings. The program listing in the appendix was written for this purpose on the Hewlett-Packard HP-32S II calculator, but it can be adapted for other scientific calculators that use Reverse Polish Notation (RPN) with minor changes (if any). The program can accept the coordinates in either dd+mm+ss (for example, $66^\circ 52' 31.5''$) or dd.dddd (for example, 66.5407°) by setting the input mode, and it automatically resolves any ambiguity.

The procedures for using the distance/bearing calculator program are as follows:

- 1) Obtain the latitudes and longitudes for the two sites. In the program, the latitude and longitude for Site A is designated A and O, respectively. For Site B, these are T and G, respectively. The equivalences with the previous expressions are:

$$A = LA_A \text{ (Site A Latitude)}$$

$$O = LO_A \text{ (Site A Longitude)}$$

$$T = LA_B \text{ (Site B Latitude)}$$

$$G = LO_B \text{ (Site B Longitude)}$$

- 2) To find the bearings, invoke program label D by pressing XEQ D. One of the initial program steps places the calculator in the DEGREE mode, so it is not necessary to do this manually.
- 3) Answer the INPUT M? prompt with 0 if the coordinates are given in the dd+mm+ss format or 1 if the coordinates are in the dd.dddd format. Press R/S.
- 4) Answer the input prompts for A, O, T and G in the appropriate format. If M = 0, input the degrees, minutes and seconds as dd.mmss (for example, enter 55° 22' 36.1" as 55.22361). If M = 1, input dd.dddd. Press R/S after each input operation. The coordinates are stored in the A-, O-, T- and G-registers in the dd.dddd format regardless of input mode.
- 5) Calculator display shows RUNNING as it makes the calculations.
- 6) When finished, the calculator displays $B_{A \rightarrow B}$ in the x-register (display) and $B_{B \rightarrow A}$ in the y-register. Bearings are in degrees with respect to true north. The program makes the necessary judgements to ensure the bearings are positive numbers between 0° and 360°.
- 7) To find the distance, invoke program label L by pressing XEQ L.
- 8) The distance between Site A and Site B is displayed in statute miles (the bearings are moved to the y-register and z-register).

The following worked examples can be used to check the programming:

Item	Site A	Site B	Site A	Site B	Site A	Site B
Location	Kotzebue	Noorvik	Kotzebue	Buckland	Red Dog Mine	Kivalina
Latitude	66° 53' 50.7"	66° 50' 03.3"	66° 53' 50.7"	65° 58' 43.1"	68.0727°	67.7259°
Longitude	162° 35' 55.7"	161° 02' 03.2"	162° 35' 55.7"	161° 07' 33.6"	162.8526°	164.5383°
$B_{A \rightarrow B}$	95.1°T		146.7°T		242.1°T	
$B_{B \rightarrow A}$		276.6°T		328.0°T		60.6°T
Distance	42.7 mi		75.4 mi		49.9 mi	

Appendix ~ Program Listing for HP 32S II programmable calculator

Note: Load program top-to-bottom, left-to-right.

LBL D	COS	LBL R	LBL Z	LBL L
DEG	RCL A	RCL L	RCL A	RCL L
SF 1	SIN	SIN	RCL T	COS
INPUT M	X	X≥0?	–	RCL A
X=0?	RCL T	GTO Y	X≥0?	COS
GTO C	TAN	X<>Y	GTO U	X
CF 1	RCL A	X≥0?	0	RCL T
LBL C	XEQ V	GTO J	180	COS
INPUT A	STO X	360	RTN	X
FS? 1	XEQ R	RTN		RCL A
→HR	STO + X		LBL U	SIN
STO A	RCL G	LBL Y	180	RCL T
INPUT O	RCL O	X<>Y	0	SIN
FS? 1	–	X≥0?	RTN	X
→HR	STO L	GTO K		+
STO O	SIN	GTO J	LBL V	ACOS
INPUT T	RCL L	RTN	COS	69.09
FS? 1	COS		X	X
→HR	RCL T	LBL J	X<>Y	
STO T	SIN	180	–	
INPUT G	X	RTN	÷	
FS? 1	RCL A		ATAN	
→HR	TAN	LBL K	RTN	
STO G	RCL T	0		
RCL O	XEQ V	RTN		
RCL G	STO Y			
–	XEQ R			
STO L	STO + Y			
X=0?	RCL Y			
GTO Z	RCL X			
SIN	RTN			
RCL L				

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