Square VLF Loop Antenna, 1.2 m Diagonal

~ Mechanical and Electrical Characteristics and Construction Details ~

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1. Dimensions

The loop antenna described here has a square shape with a diagonal length of 1.207 m. The loop is made from fiberglass framework and marine plywood braces and consists of 64 ±1 turns of 18 AWG coated magnet wire.

The width of a square in terms of its diagonal length is

$$W = \frac{\sqrt{2}}{2} \cdot l \qquad \textbf{Eq. (1.1)}$$

where
W = square width
l = diagonal length (1.207 m)

The perimeter length of a square is

$$p = 4 \cdot W \qquad \textbf{Eq. (1.2)}$$

where
p = perimeter length

Therefore, for the loop in question

$$W = \frac{\sqrt{2}}{2} \cdot 1.207 = 0.853 \text{ m} = 85.3 \text{ cm}$$

$$p = 4 \cdot 0.853 = 3.412 \text{ m}$$

The enclosed area of a square is

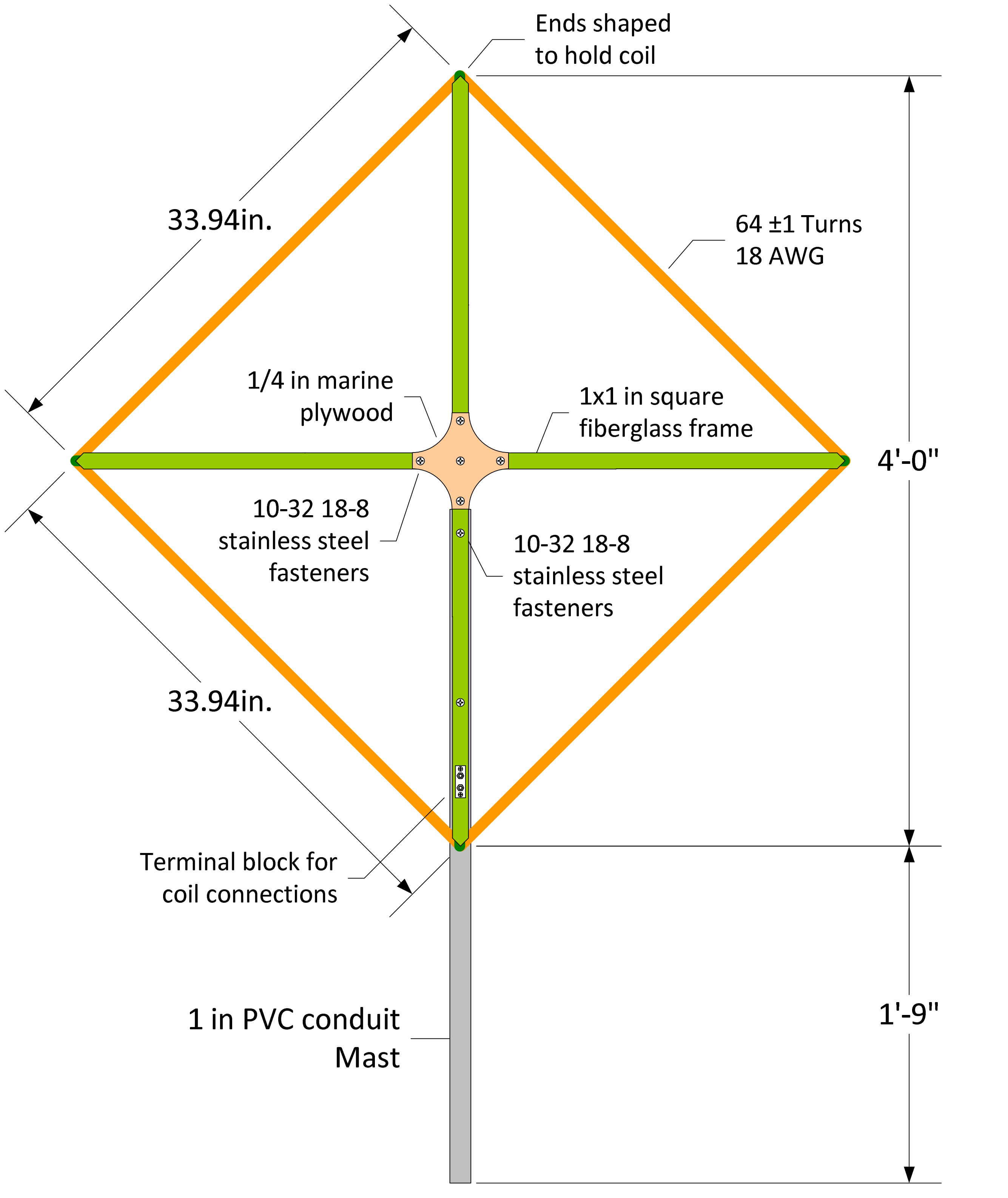
$$A = W^2 = (0.0.853)^2 = 0.733 \text{ m}^2 \qquad \textbf{Eq. (1.3)}$$

where
A = enclosed area of square

The loop is wound with 18 AWG magnet wire, which has a coated nominal diameter of approximately 0.0424 in. = 1.077 mm = 0.1077 cm and uncoated nominal diameter of 0.0403 in. = 1.0236 mm = 0.1024 cm.

The total wire length is
$Length = p \cdot T = 3.412 \cdot 64 = 218.4$ m

where
T = Number of turns


Ends shaped
to hold coil
33.94in.
64 ±1 Turns
18 AWG
1/4 in marine
plywood
1x1 in square
fiberglass frame
4'-0"
10-32 18-8
stainless steel
fasteners
10-32 18-8
stainless steel
fasteners
33.94in.
Terminal block for
coil connections
1 in PVC conduit
Mast
1'-9"

2. Inductance

Four methods are investigated for calculating the loop inductance. All methods are estimates and none perfectly fit the actual antenna construction but method 1 is the closest. The coil cross-section on the actual antenna is more or less rectangular, but two of the methods assume a flat, single-layer cross-section. The formulas used here are based on a 1937 National Bureau of Standards document in which the CGS unit system is used. To avoid units conversion errors, all calculations given here use the same units. Also included are actual measurements for comparison with calculations.

Method 1 (Note: This method assumes a multi-layer coil with rectangular cross-section)

The approximate inductance in μH of a square coil with rectangular cross-section is given by[1]

$$L = 0.01257 \cdot a \cdot n^2 \left[2.303 \cdot \left(1 + \frac{b^2}{32 \cdot a^2} + \frac{c^2}{96 \cdot a^2} \right) \log\left(\frac{8 \cdot a}{d} \right) - y_1 + \frac{b^2}{16 \cdot a^2} \cdot y_2 \right] \qquad \textbf{Eq. (2.1)}$$

where

a	= Average of inscribed and circumscribed radii, $r \cdot \cos^2\left(\frac{\pi}{2 \cdot N}\right)$ (cm)
b	= axial dimension of the coil cross-section (cm)
c	= radial dimension of the coil cross-section (cm)
d	= diagonal of the cross-section, $\sqrt{b^2 + c^2}$ (cm)
n	= number of turns
y_1	= value from Table 14, pg 285 of reference based on b/c
y_2	= value from Table 14, pg 285 of reference based on c/b

The following values apply:

a	= 51.5 cm
b	= 2.22 cm
c	= 0.2154 cm
d	= 2.23 cm
n	= 64
b/c	= 10.3
c/b	= 0.097
y_1	= 0.5898
y_2	= 0.1317

Substituting the above values, the calculated inductance is

$$L = 0.01257 \cdot 51.5 \cdot 64^2 \left[2.303 \cdot \left(1 + \frac{2.22^2}{32 \cdot 51.5^2} + \frac{0.2154^2}{96 \cdot 51.5^2} \right) \cdot \log\left(\frac{8 \cdot 51.5}{2.23} \right) - 0.5898 + \frac{2.22^2}{16 \cdot 51.5^2} \cdot 0.1317 \right] =$$

12 278 μH = **12.3 mH**

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[1] Eq. 157, pg 257, Circular C74, Radio Instruments and Measurements, US Department of Commerce, National Bureau of Standards, 1937

Method 2 (Note: This method is similar to Method 1 but assumes a single coil layer)

The approximate inductance in μH of a single-layer square coil is given by[2]

$$L = 0.008 \cdot a \cdot n^2 \left[2.303 \cdot \log\left(\frac{a}{b}\right) + 0.2231 \cdot \frac{b}{a} + 0.726 \right] - 0.008 \cdot a \cdot n \cdot \left[A + B \right] \qquad \textbf{Eq. (2.2)}$$

where
a = width of square measured between centers of the cross-section (cm)
b = length of coil (cm)
n = number of turns
D = coil winding pitch (cm) (0.1077 cm)
d = diameter of bare wire (cm) (0.1024 cm)
A = function of Table 11 in NBS reference document
B = function of Table 12 in NBS reference document

The coil width is a = W = 85.3 cm, length is $b = 22.2$ mm = 2.22 cm and the number of turns n = 64. A is based on the ratio $d/D = 0.1024/0.1077 = 0.95$ in Table 11 on page 284 of the reference and B is based on the number of turns n = 64 in Table 12 on page 284 of the reference. Values are
A = 0.506
B = 0.322

Substituting the above values, the calculated inductance is

$$L = 0.008 \cdot 85.3 \cdot 64^2 \left[2.303 \cdot \log\left(\frac{85.3}{2.22}\right) + 0.2231 \cdot \frac{2.22}{85.3} + 0.726 \right] - 0.008 \cdot 85.3 \cdot 64 \cdot \left[0.506 + 0.322 \right] =$$

12 210 μH = **12.2 mH**

Method 3 (Note: This method assumes a rectangular cross-section)

The approximate inductance in μH of a square coil with rectangular cross-section is given by[3]

$$L = 0.008 \cdot a \cdot n^2 \left[2.303 \cdot \log\left(\frac{a}{b+c}\right) + 0.2235 \cdot \frac{b+c}{a} + 0.726 \right] \qquad \textbf{Eq. (2.3)}$$

where
a = width of square measured between centers of the cross-section (cm)
b = length of coil (cm)
c = depth of coil (cm)
n = number of turns

The coil width is a = W = 85.3 cm, length is $b = 22.2$ mm = 2.22 cm, c = 2.154 mm = 0.2154 cm and the number of turns n = 64.

[2] Eq. 165, pg 264, Circular C74, Radio Instruments and Measurements, US Department of Commerce, National Bureau of Standards, 1937
[3] Eq. 163, pg 263, Circular C74, Radio Instruments and Measurements, US Department of Commerce, National Bureau of Standards, 1937

Substituting the above values, the calculated inductance is

$$L = 0.008 \cdot 85.3 \cdot 64^2 \left[2.303 \cdot \log\left(\frac{85.3}{2.22 + 0.2154} \right) + 0.2231 \cdot \frac{2.22 + 0.2154}{85.3} + 0.726 \right] = 11\,990\ \mu\text{H} = \mathbf{12.0\ mH}$$

Method 4 (Note: this method assumes a single-layer coil of length b of polygon shape)

The approximate inductance in μH of a polygon is given by[4]

$$L \approx \frac{0.03948 \cdot a^2 \cdot n^2}{b} \cdot K \ \ \mu\text{H} \qquad \textbf{Eq. (2.4)}$$

where

a = Average of inscribed and circumscribed radii, $r \cdot \cos^2\left(\frac{\pi}{2 \cdot N} \right)$ (cm) **Eq. (2.5)**

r = radius of circumscribed circle (cm)

N = number of sides (4 for a square)

n = number of turns

b = length of coil, or $n \cdot d$ (cm)

d = distance between turn centers = wire diameter for close spacing (cm) (0.1077 cm for 18 AWG coated wire)

K = function of $2 \cdot a / b$ from table 10 in the reference NBS document

The radius of a circumscribed circle for a square is

$$r = \frac{W}{\sqrt{2}} = \frac{l}{2} = \frac{1.207}{2} = 0.6033 \text{ m} = 60.33 \text{ cm} \qquad \textbf{Eq. (2.6)}$$

and the average of the inscribed and circumscribed radii is

$$a = 60.33 \cdot \cos^2\left(\frac{\pi}{2 \cdot 4} \right) = 51.5 \text{ cm}$$

The estimated coil length for single-layer flat winding is $b = n \cdot d = 62 \cdot 0.1077 = 6.68$ cm. K is based on $2 \cdot a / b = 2 \cdot 51.5 / 6.68 = 15.42$ and is found in Table 10, page 283 of the reference NBS document by interpolation, or

K = 0.1498.

Substituting the above values, the calculated inductance is

$$L \approx \frac{0.03948 \cdot 51.5^2 \cdot 64^2}{6.68} \cdot 0.1498 = 9\,618\ \mu\text{H} = \mathbf{9.6\ mH}$$

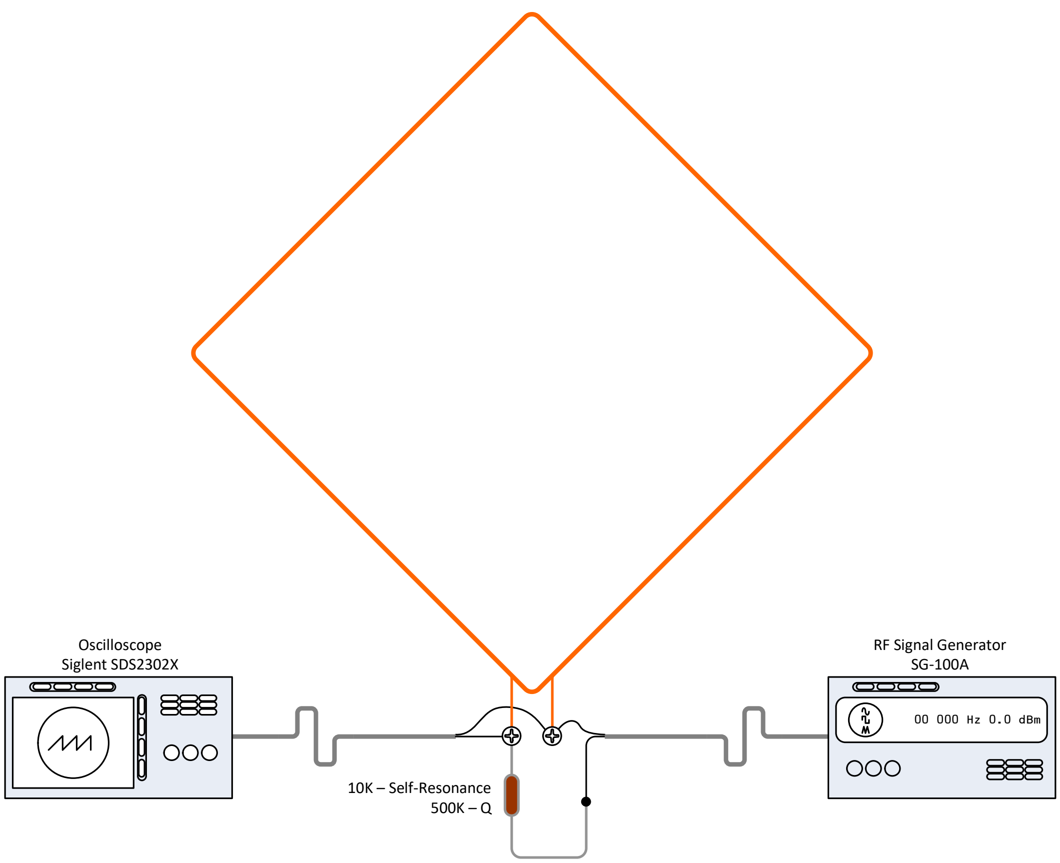
[4] Eq. 153, pg 252, Circular C74, Radio Instruments and Measurements, US Department of Commerce, National Bureau of Standards, 1937

3. Measurements

Measurement date: 10 August 2018
Measured inductance at 15.5 °C:
DM4070: 12.29 mH
Peak LCR45: 12.24 mH at 1 kHz
Peak LCR45: 13.65 mH at 15 kHz
Peak LCR45: 680 uH at 200 kHz

Measurement date: 2 September 2018 (on-site)
Measured inductance at 15.0 °C:
Keysight U1733C: 12.270 mH, Q = 16.7 at 1 kHz
Keysight U1733C: 12.841 mH, Q = 69.1 at 10 kHz

Additional measurements setup:



Measured self-resonance frequency: 46.065 kHz

Measured dc resistance at 20.6 °C: 4.535 ohms

Calculated dc resistance: 6.386 ohms/1000 ft x 3.412 m x 64 / 0.3048 = 4.5751 ohms at 20 °C
Calculated distributed capacitance, C_d , based on self-resonance: 1020 pF

Peak voltage at resonance: 0.4118 V, 6 dBV reduction: 0.4118 V x 0.707 = 0.2911 V
Frequencies at 6 dBV reduction (–3 dB power): 38 173 and 55 523 Hz, Frequency change = 3 dB power bandwidth: 17 350 Hz

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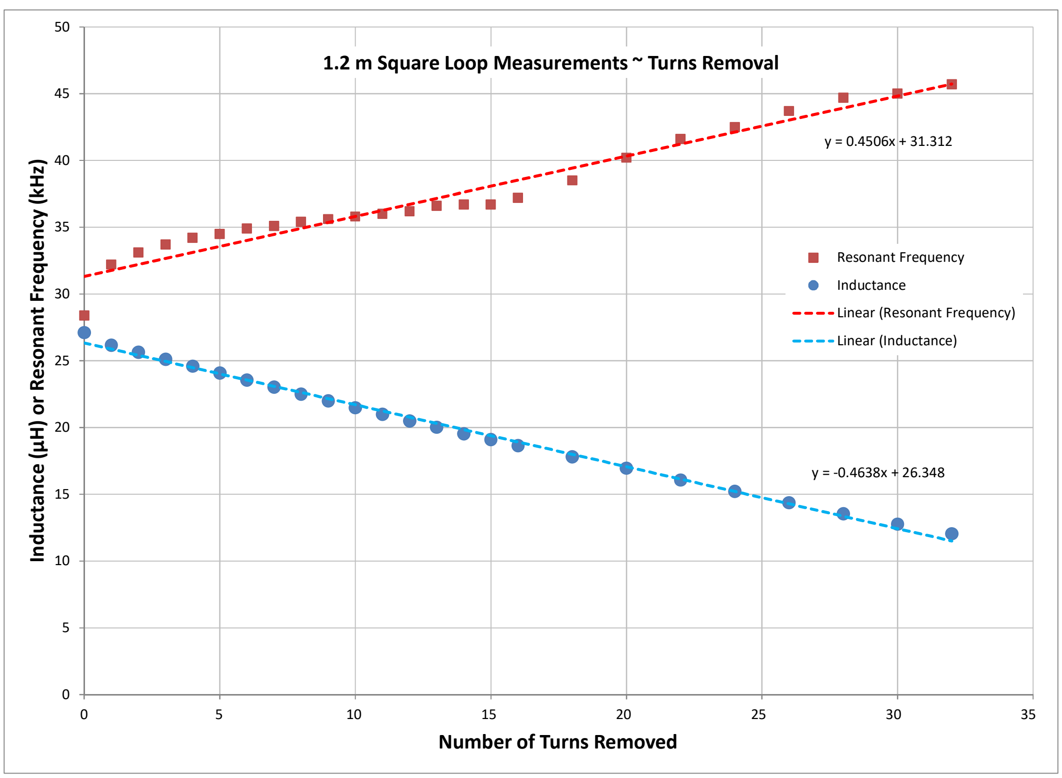
The loop originally was built in late 2009 using a 1200 ft spool of 18 AWG coated magnet wire resulting in 96 ±1 turns. The average individual winding length was approximately 11.25 ft (3.43 m). After construction I made a series of inductance and self-resonant frequency measurements as follows:

Instrument: TH2811D, Temperature: 20.6 °C

Test Frequency (Hz)	Inductance (µH)	Q	R (ohms)
100	27.126	2.4942	6.8364
120	27.117	2.9865	6.8438
1000	27.116	23.9	7.14
10000	30.581	70	27.6
Calculated	27.625	N/A	6.8627

I measured the self-resonant frequency at 28.4 kHz but this was much too low for applications as a tuned loop for VLF and LF applications. I then removed one turn at a time and measured the inductance and self-resonant frequency after each turn was removed. I removed 32 turns to achieve the desired self-resonant frequency of 45.6 kHz. See chart below.



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4. Open circuit voltage

From Faraday's law of induction

$$V = -\frac{d\varphi(t)}{dt} \qquad \textbf{Eq. (4.1)}$$

where

V = open circuit rms voltage (V)

φ(t) = magnetic flux (weber = $v \cdot s$)

t = time (s)

Therefore, an induced voltage appears across the terminals of a circuit immersed in a changing magnetic field. If the circuit consists of an electrically small air core loop antenna with n turns, the voltages in the turns are additive. Note: An electrically small loop antenna has circumference much less than a wavelength. For any frequency < 300 kHz one wavelength in free space is > 1,000 m, and the circumference of any practical loop antenna is much smaller.

$$V = -n \cdot \frac{d\phi(t)}{dt} \qquad \textbf{Eq. (4.2)}$$

The magnetic flux is related to the time varying magnetic induction by

$$\phi(t) = B(t) \cdot A_e \cdot \cos(\theta) = B \cdot \cos(\omega \cdot t) \cdot A_e \cdot \cos(\theta) \qquad \textbf{Eq. (4.3)}$$

where

B(t) = magnetic induction (tesla, T = $v \cdot s / m^2$)

B = rms magnetic induction (T)

A_e = Area of equivalent circular loop with radius a, $\pi \cdot a^2$ (m)

ω = radian frequency ($2 \cdot \pi \cdot f$, radians/s)

f = frequency (Hz)

ϑ = angle between magnetic field lines and normal of loop frame (radians)

a = Average of inscribed and circumscribed radii, $r \cdot \cos^2\left(\frac{\pi}{2 \cdot N}\right)$ (m) **Eq. (4.4)**

r = radius of circumscribed circle, d/2 (m)

N = number of sides (4 for an square)

The radius of a circumscribed circle for a square is 1/2 of the square's diagonal dimension. For the square loop in question, radius $r = 1.207/2 = 0.6035$ m, and from eq. (4.4) the average of inscribed and circumscribed radii is

$$a = 0.6035 \cdot \cos^2\left(\frac{\pi}{2 \cdot 4}\right) = 0.5151 \text{ m}$$

Therefore, for the square loop in question, the equivalent area $A_e = \pi \cdot a^2 = \pi \cdot 0.5151^2 = 0.8336\,\mathrm{m}^2$

Differentiating Eq. (4.3) gives

$$\frac{d\phi(t)}{dt} = \frac{dB(t)}{dt} \cdot A_e \cdot \cos(\theta) = B \cdot A_e \cdot \cos(\theta) \cdot \frac{d\cos(\omega \cdot t)}{dt} = -B \cdot A_e \cdot \cos(\theta) \cdot \omega \cdot \sin(\omega \cdot t) \qquad \textbf{Eq. (4.5)}$$

Substituting Eq. (4.5) in Eq. (4.1), the open circuit voltage across the loop terminal is

$$V(t) = 2 \cdot \pi \cdot f \cdot n \cdot A_e \cdot B \cdot \cos(\theta) \cdot \sin(\omega \cdot t)$$

and the open circuit rms voltage is

$$V = 2 \cdot \pi \cdot f \cdot n \cdot A_e \cdot B \cdot \cos(\theta) \qquad \textbf{Eq. (4.6)}$$

The above expression indicates the loop antenna responds to the magnetic field component (magnetic induction or flux density, B) of a signal and converts it to a voltage at the antenna terminals.

The voltage at the terminals is related to the electric field strength, E, by

$$V = h_e \cdot E \qquad \textbf{Eq. (4.7)}$$

where
h_e = effective antenna height (m)
E = rms electric field strength (V/m)

The relationship between the electric field strength and magnetic induction is

$$E = c \cdot B \qquad \textbf{Eq. (4.8)}$$

where
c = speed of light in free space ($3 \cdot 10^8$ m/s)

Substituting Eq. (4.6), (4.7) and (4.8), the effective height of an air-core loop is

$$h_e = \frac{2 \cdot \pi \cdot f \cdot n \cdot A_e \cdot \cos(\theta)}{c} = \frac{2 \cdot \pi \cdot n \cdot A_e \cdot \cos(\theta)}{\lambda} \qquad \textbf{Eq. (4.9)}$$

where
λ = wavelength (m)

It is seen from eq. (4.7) that, for a given electric field strength, the rms voltage at the loop terminals is proportional to the effective height. Equivalently, from eq. (4.9) it is proportional to the frequency, number of turns and loop equivalent area. Note that the effective height is not related to the physical height of the loop – it relates the field strength to open circuit voltage. For a given frequency the effective height can be increased by increasing the number of turns or loop area.

For the case where the loop frame is parallel to the propagation direction and the magnetic field is normal to the propagation direction, in which case θ = 0 deg. = 0 radians, eq. (4.9) becomes

$$h_e = \frac{2 \cdot \pi \cdot n \cdot A_e \cdot f}{c}$$ **Eq. (4.10)**

If the frequency is 24 kHz, the effective height of the example antenna is

$$h_e = \frac{2 \cdot \pi \cdot n \cdot A_e \cdot f}{c} = \frac{2 \cdot \pi \cdot 64 \cdot 0.8336 \cdot 24 \cdot 10^3}{3 \cdot 10^8} = 0.0268 \text{ m}$$

If the field strength is 500 μv/m at 24 kHz, the open circuit (unloaded) loop terminal voltage for the example loop is

$$V = h_e \cdot E = 0.0268 \cdot 500 = 13.4 \ \mu\text{V}$$

In air and free space, the magnetic field strength and magnetic induction are related by

$$H = \frac{B}{\mu_0}$$ **Eq. (4.11)**

where
H = rms magnetic field strength (a/m)
μ_0 = permeability of air or free space ($4\pi \times 10^{-7}$ henry/m)

Therefore, when the loop frame is parallel to the line of signal propagation, the rms loop terminal voltage is maximum and in term of the magnetic field strength is given by

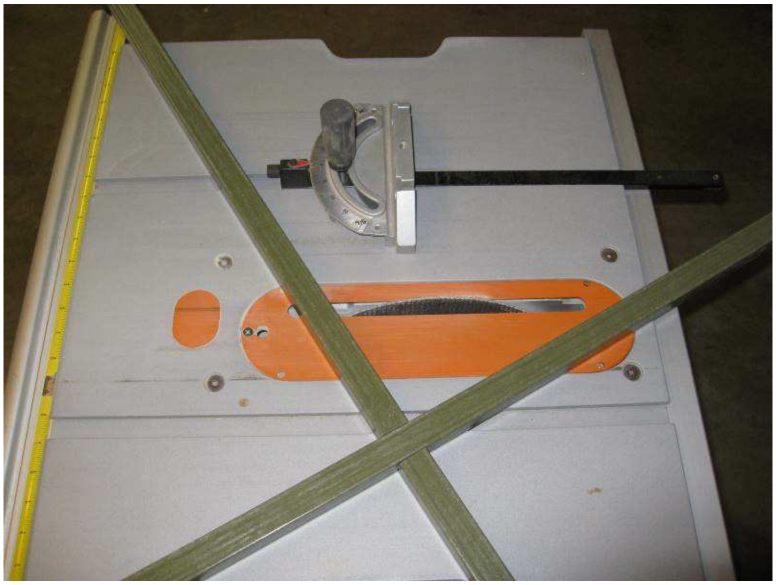
$$V_{max} = 2 \cdot \pi \cdot \mu_0 \cdot n \cdot A \cdot f \cdot H$$ **Eq. (4.12)**

5. Construction images

The loop was constructed between 28 November and 6 December 2009.

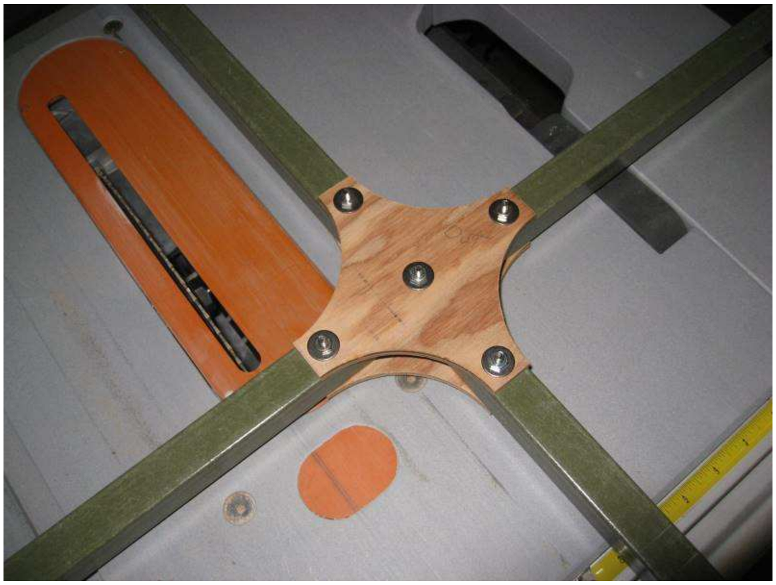
1" square fiberglass tube was used for the loop frame or cross-braces. The loop dimensions were determined by the length of the 8 ft tube I had on-hand; it was simply cut in half. The braces were cut on a table saw with an abrasive blade used for cutting metal. The center of each brace was then notched on the saw so they fit together at the center.

The ends of each brace were cut at 45 angle on the table saw and lightly sanded. The notched ends hold the wire in place.



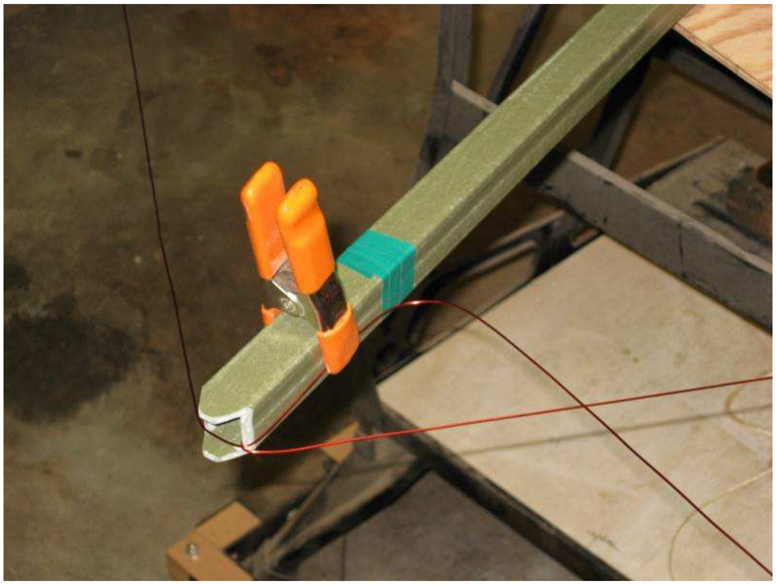
The two braces fit together at their centers.

Reinforcements were cut from 1/4” marine plywood with a 6” hole saw on a drill press and then cut into pieces on a table saw.

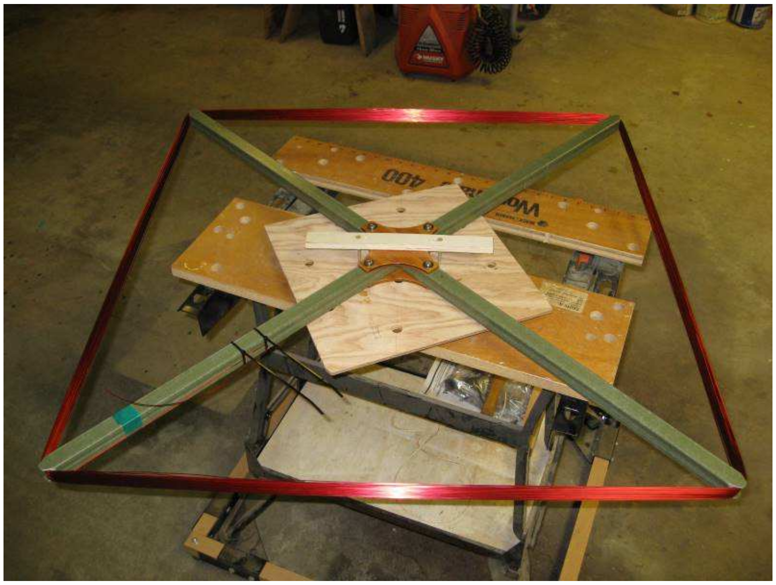


After sanding the reinforcements, they were clamped to the braces to ensure alignment and drilled with a 3/16 in brad point drill. The fasteners are 18-8 stainless steel 10-32 machine screws, washers and nuts. After this image was taken, the wood pieces were removed and painted with urethane spar varnish.

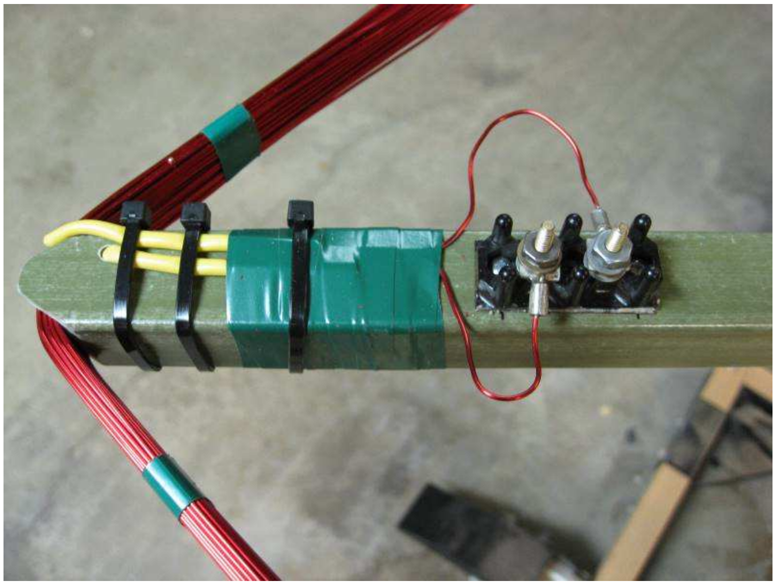
After parts were permanently assembled, the loop frame was clamped to a wood plate with a mandrel on then placed on a shop table for winding. The spool of magnet wire was placed on another mandrel clamped to an adjacent table.



The starting end of the 18 AWG coated magnet wire was clamped to the frame and then the frame was slowly rotated.

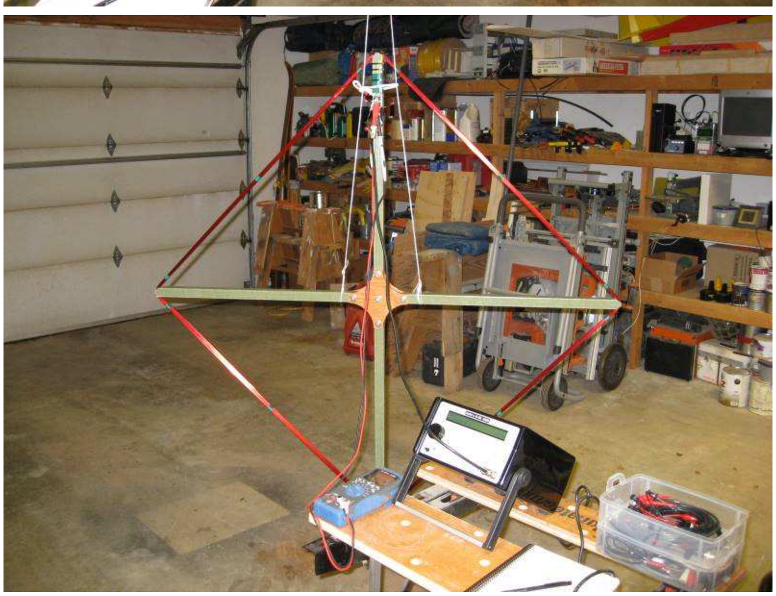


Completed loop antenna with 96 turns and about 1100 ft of wire. 32 turns were later removed as discussed in the measurements section leaving about 720 ft on completed antenna.



View of the wire terminations on the loop frame. The wire is protected by insulating sleeves and tape where clamped to the frame. The terminal block was later installed in a plastic enclosure.

The loop was hung from the shop ceiling for measurements. This view shows the TH2811D LCR meter used for the inductance and resistance measurements in 2009.



For self-resonant frequency measurements in 2009, a Telulex SG-100A signal generator and Tektronix TDS 2022B oscilloscope was used. The signal generator 50 ohm output was isolated from the loop with 10 kohm and 500 kohm resistors. Measurements were duplicated in 2018 using this setup (but with a different oscilloscope).

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0.1 (Substituted actual values for calculations, 06 Aug 2018)
0.2 (Added measurements, 11 Aug 2018)
0.3 (Updated calculations, 15 Aug 2018)
0.4 (Completed 1st draft for distribution, 20 Aug 2018)
0.5 (Added original construction images and data, 21 Aug 2018)
0.6 (Added open circuit voltage calculations, 23 Aug 2018)
0.7 (Added U1733C on-site measurements, 04 Sep 2018)
0.8 (Minor updates to construction section, 07 Sep 2018)
0.9 (Added wire length, 12 Sep 2018)

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