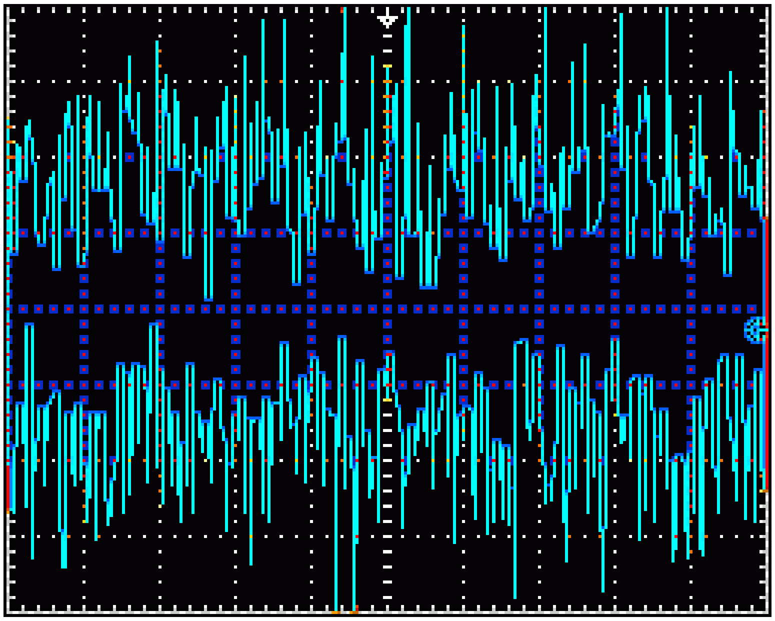
Noise Tutorial
Part I ~ Noise Concepts



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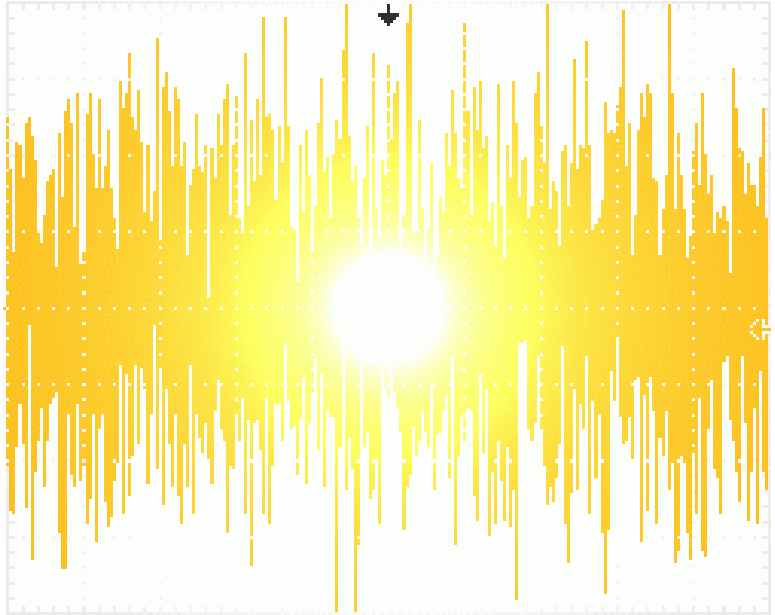

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Abstract: With the exception of some solar radio bursts, the extraterrestrial emissions received on Earth's surface are very weak. Noise places a limit on the minimum detection capabilities of a radio telescope and may mask or corrupt these weak emissions. An understanding of noise and its measurement will help observers minimize its effects. This paper is a tutorial and includes six parts.

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Noise Tutorial I ~ Noise Concepts

Part I ~ Noise Concepts

1-1. Introduction

Noise is a random electrical waveform that can obscure the desired radio emissions and signals. The random nature of a noise voltage waveform means 1) it contains no predictable periodic frequency components and 2) its amplitude at any future time is unpredictable. Unlike periodic signals, such as manmade broadcast transmissions that consist of one or more discrete spectral lines corresponding to their frequency components, random noise has a spectrum that is a continuous function of frequency and contains no discrete line components (figure 1-1).

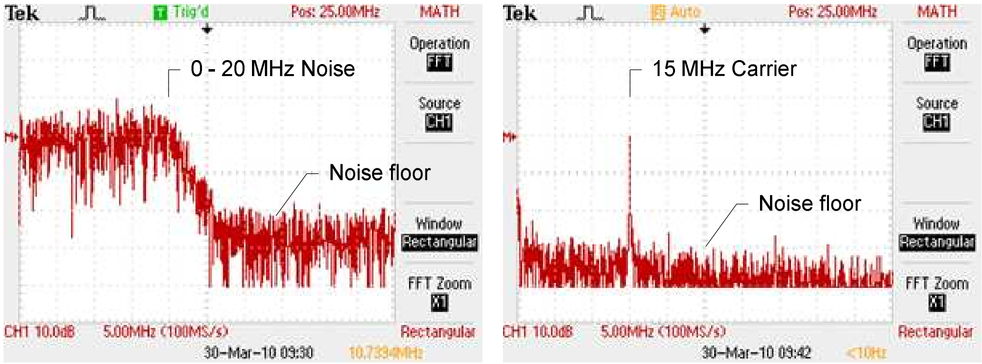


Fig. 1-1 ~ Noise and periodic signal spectrums. Left: The noise spectrum is produced by a 20 MHz random noise generator and displayed on an oscilloscope. The noise extends continuously four divisions (5 MHz/div) from the left scale and drops off due to the lowpass filter in the generator. Right: The periodic signal is an unmodulated 15 MHz carrier wave (CW) indicated as a single spectral line located three divisions from the left scale.

1-2. Basic noise sources

An important source of noise is the constant agitation of matter at molecular and atomic levels. The molecules vibrate about their position in solids or collide with each other in gases. These are called thermal agitations because they are related to temperature. A resistor or copper wire has conduction electrons that are free to wander randomly throughout the material volume. These electrons as well as positive ions also present in the material are uniformly distributed, and the entire structure is electrically neutral. However, because of the random motions, there are statistical fluctuations away from the neutral state. A very large number of charges are involved and, occasionally, the charge distributions will not be uniform and a voltage difference will appear across the conductor terminals. This random voltage is erratic and unpredictable and is called resistor thermal noise, or just *thermal noise* (also called Johnson noise).

Thermal noise is proportional to the temperature of the material and the value of its resistance. It is present at the resistor terminals whether or not it is connected to anything. Thermal resistor noise is zero for a perfect conductor and any conductor at a temperature of absolute zero (0 kelvin). In noise analysis and in this paper, all temperatures are in kelvin (K). Temperatures in kelvin are related to temperatures in celsius by

$$\begin{aligned} T(K) &= T(^{\circ}C) + 273.15 \\ T(^{\circ}C) &= T(K) - 273.15 \end{aligned} \quad \text{(the conversion constant is rounded to 273 in ordinary work)} \qquad (1\text{-}1)$$

Another type of noise is caused by the flow of current across semiconductor junctions in diodes and transistors. The charge carriers, electrons or holes, enter the junction from one side, drift or are accelerated across the junction, and are collected on the other side. The average current across the junction determines the average time interval between two successive carriers that enter the junction. However, there are random fluctuations in the movement, giving rise to a type of noise called *shot noise*. Shot noise also is caused by the random electron emissions from a heated surface, such as the filament in a vacuum tube or other thermionic device. Resistors and other electronic components in radio telescopes are called *internal* noise sources.

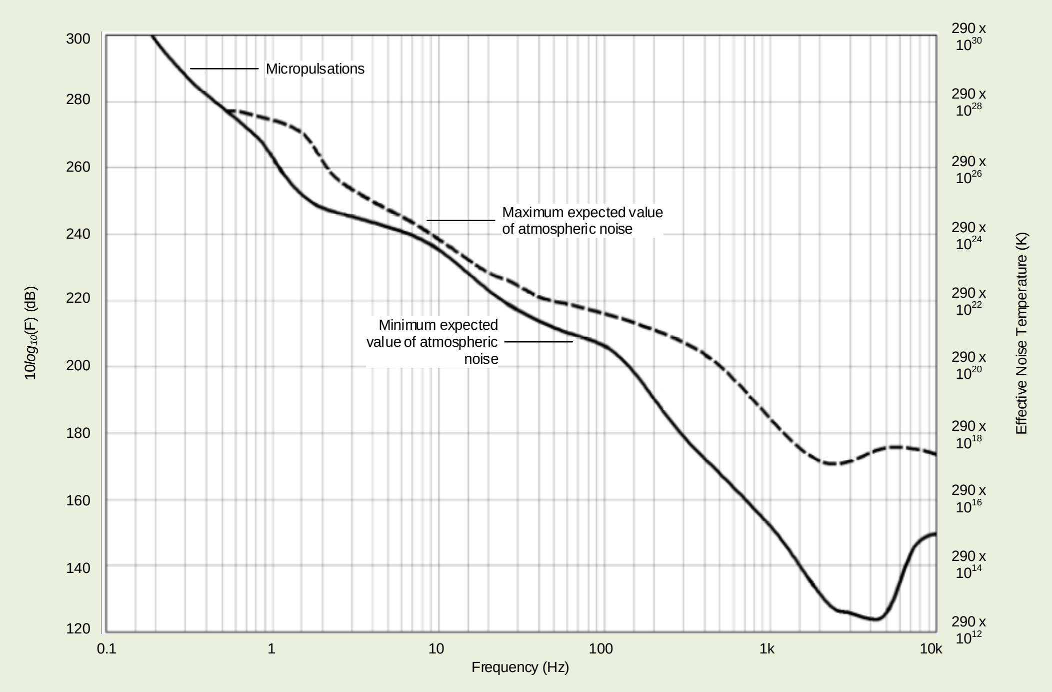
Noise sources *external* to telescopes also are very important. They consist of sky noise, manmade noise and test and measurement noise. The first two are generalized in a set of charts that extend from 0.1 Hz to 100 GHz (figure 1-2). These charts are provided for reference as we discuss noise technical parameters.

- *Sky noise* due to electrical storms, solar and geomagnetic storms, cosmic radio noise and atmospheric absorption. When these external noise sources interfere with the desired emissions, they are called electromagnetic interference (EMI), radio frequency interference (RFI) or just interference. On the other hand, we may be actually investigating certain types of sky noise, including radio emissions from the Sun and Jupiter, neutral hydrogen emissions or cosmic microwave background (CMB) radiation, in which case it obviously is not considered interference. Some of these emissions are due to synchrotron radiation (see sidebar) from very large numbers of charged particles in outer space. When large numbers of independent sources are combined the resulting emissions have random and very noise-like characteristics. Also, during their propagation these emissions are randomly modulated by inter-stellar plasma and the Sun's plasma (solar wind). Other sources of sky noise may mask the specific emissions being investigated. For example, lightning can mimic certain types of Jupiter emissions and the CMB may be strong enough to mask interesting emissions from specific extraterrestrial sources in the same direction. For our purpose, sky noise is due to natural phenomena.

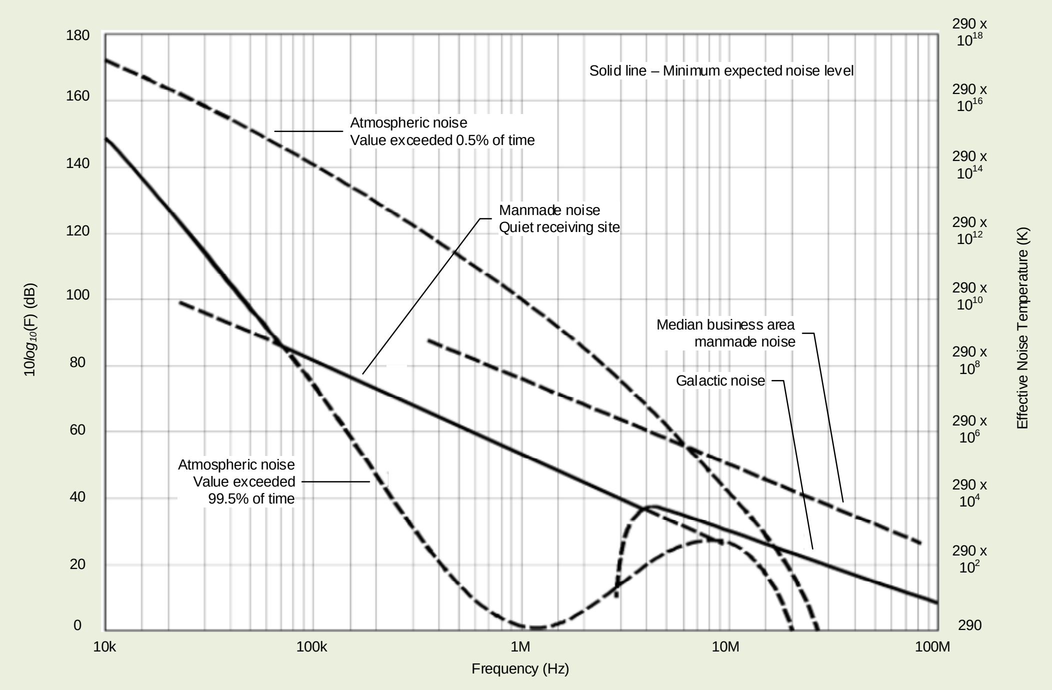
Synchrotron radiation is electromagnetic radiation generated by charged particles moving at speeds approaching the speed of light (relativistic speeds) in spiral orbits around magnetic field lines.

- *Manmade noise* due to ignition systems, electric motors and relays, arcing and corona discharges in electrical power systems and transmission lines, and consumer electrical and electronic equipment such as LED lighting, televisions, radio receivers and transmitters, microwave ovens, dimmer switches, ac adapter power supplies, coffee grinders and personal computers. These noise sources may be close by or far away and can be from terrestrial or satellite transmitters. Except at locations dedicated for the purpose, radio astronomers have little or no control over many manmade noise sources and must find ways to minimize their effects or contend with them.

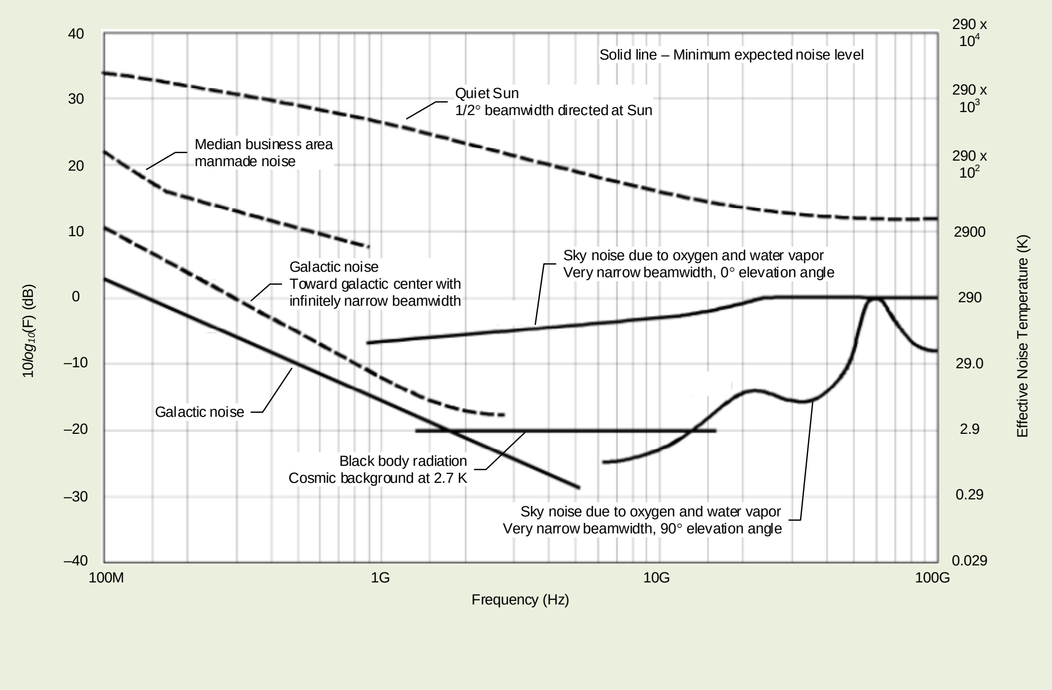
- *Test and measurement noise* from calibrated noise sources (figure 1-3). These use semiconductor diodes or special thermionic vacuum tubes that are built into or temporarily connected to the radio telescope receiver or antenna systems for calibration purposes and are under control of the radio telescope operator.



(a) 0.1 Hz to 10 kHz: There is very little seasonal, diurnal or geographic variation. The variation in the range of 5 to 10 kHz is due to the variability of the Earth-ionosphere waveguide mode cutoff and the resulting propagation of atmospheric and other noise at those frequencies



(b) 10 kHz to 100 MHz: The atmospheric noise values takes into account all times of day, seasons and the entire Earth's surface. The curve for galactic noise above about 5 MHz is greatly affected by the ionospheric absorption that can extend to above 20 MHz. For directional antennas in the HF range (3 - 30 MHz), studies of atmospheric noise show that there can be ±5 dB variation around the indicated noise factor depending on direction, frequency and geographical location



(c) 100 MHz to 100 GHz: The minimum galactic noise from a narrow-beam antenna pointed at the galactic pole is 3 dB below the solid curve shown. The sky noise due to oxygen and water vapor absorption is quite variable above about 1 GHz.

Fig. 1-2 ~ Sky and manmade noise across the radio bands from 0.1 Hz to 100 GHz. Except as noted, the curves are for omni-directional antennas. The left scale on all plots is in terms of noise factor (in dB) and the right scale is in terms of noise temperature (in kelvin). (Plots adapted from: [ITU-R P372.8], used with permission)

Fig. 1-3 ~ Calibrated noise sources. Left-to-right: Hewlett-Packard 346D (ENR 21 dB), Renz RQ6 with 20 dB attenuator (ENR 55 dB without attenuator), RF Associates RF-2050S (Noise temperature switchable 43 thousand to 1.4 million kelvin), RF Design RFD2305 with 10 dB attenuator (ENR 5 dB with 10 dB attenuator). ENR (excess noise ratio) is discussed in Part V.

All attempts to receive extraterrestrial emissions involve noise. Most extraterrestrial radio sources are physically very large (but small in angular size when viewed from Earth) and their radiation involves a very large number of statistically independent radio sources – atoms, electrons, ions and molecules. The resulting emissions are noise-like and in many situations are indistinguishable from noise due to manmade radio frequency interference. When these emissions arrive at a radio telescope antenna they are accompanied by interfering noise, which may be stronger than the desired emissions. Also, each part of the receiving system and each receiver stage introduce additional interfering noise.

We can describe random noise only in terms of its average properties; an instantaneous measurement means little. Therefore, noise measurements are made on an average basis over some finite measurement time. The two most important properties of noise are amplitude (voltage or power) and spectrum (frequency distribution). Although we cannot predict the noise voltage amplitude at any instant, we can predict the amplitude range.

We often are interested in the noise power, which is proportional to the square of the noise voltage. Because an important property of noise is its spectrum, we cannot properly discuss noise power of a device without knowing the associated bandwidth. In radio discussions it is convenient to normalize noise power to a bandwidth of 1 Hz, for example, watts per hertz or milliwatts per hertz. This representation is called *noise power density*. We will discuss these units of measure and their logarithmic equivalents later.

1-3. Noise amplitude

Because noise is a random process, it is analyzed on a statistical basis. The noise amplitude prediction probability is given by a function called the *amplitude probability density distribution p(v)*. When multiplied by a small voltage increment *dv*, the amplitude probability density distribution function gives the probability that, at any given instant, the voltage lies in the interval v to *v + dv*. The amplitude density distribution can take on many forms but, as will be seen, only one form is of most interest to us. Since the noise voltage, *v*, must exist at some value, the total probability over an infinite range is 1 (100%). In other words, the sum of all the individual products of p(v) and dv throughout the range of p(v) will be

$$\int_{-\infty}^{+\infty} p(v)dv = 1 \qquad (1\text{-}2)$$

We also are interested in determining the probability that the noise voltage amplitude will fall below some given value. The sum of the amplitude density distribution over part of the voltage amplitude range is called the *cumulative amplitude probability distribution P(v)* and is defined as

$$P\left(v\right) = \int_{-\infty}^{v} p(v)dv \qquad (1\text{-}3)$$

P(v) also is called the *cumulative probability* and is the probability that a noise voltage lies below the voltage *v* at any given time. It is the area under the *p(v)* curve between the extreme negative value (–∞)

and *v*. *P(v)* varies from 0 to 1. When *P(v)* is 0, there is 0% chance the voltage is below *v*, and when *P(v)* is 0.5 there is a 50% chance the voltage is below *v*, and so on. We cannot say what the values will be at any instant, only that there is a calculated probability it will be below that value.

Similarly, we are interested in the probability that the noise voltage amplitude will fall within a given range, say *v1* and *v2*. In this case,

$$P(v) = \int_{-\infty}^{v1} p(v)dv - \int_{-\infty}^{v2} p(v)dv \qquad (1\text{-}4)$$

When we analyze noise and its effects, we must make some assumptions. One is that the statistical averages of the noise, such as its amplitude distribution and spectrum, do not change over time (the noise is said to be *stationary*, a property analogous to oscillator stability). Also, we assume the probability density of the noise is Gaussian; that is the noise conforms to a particular mathematical function called the Gaussian, or normal, distribution. The normal distribution describes the probability that a particular amplitude will be exceeded (or not exceeded) in any given measurement period. When plotted the normal distribution yields the familiar bell-shaped curve (Fig. 1-3). Other distribution functions exist, but it has been determined both theoretically and experimentally that thermal and shot noises and many other conditions we encounter in radio astronomy are Gaussian, so we will concentrate on the Gaussian distribution. It should be noted that many types of interfering noise are not Gaussian. They have strong periodic components and thus do not meet the definition of noise for our purposes. Manmade radio frequency interference is an example of non-Gaussian noise.

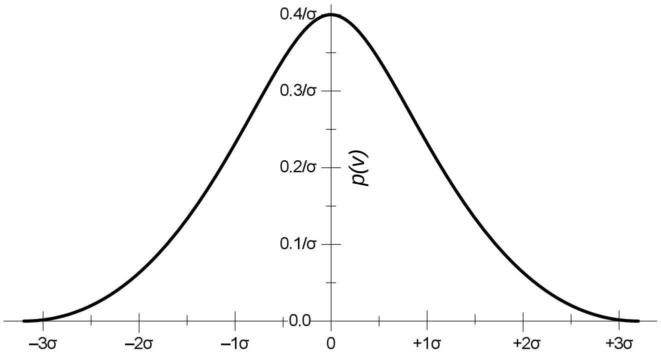


Fig. 1-3 ~ Gaussian distribution (bell-shaped or normal distribution) curve. Almost 99% of the area under this curve lies between -3σ and +3σ.

For the Gaussian amplitude density distribution

$$p(v) = \left[\frac{1}{\sigma \cdot \sqrt{2 \cdot \pi}}\right] \cdot e^{-\frac{v^2}{2 \cdot \sigma^2}} \qquad (1\text{-}5)$$

and for the Gaussian amplitude distribution

$$P(v) = \frac{1}{2}\left[1 + erf\left(\frac{v}{\sigma \cdot \sqrt{2}}\right)\right] \tag{1-6}$$

where

σ root-mean-square (rms) noise voltage, also called standard deviation

erf error function

The **error function** is a special function that frequently occurs in the mathematics of probability and statistics and is determined by integrating the Gaussian amplitude density distribution function.

There is no theoretical upper amplitude limit that noise could reach, but very high amplitudes have very low probability, and real electronic systems have practical amplitude limits. If a large number of instantaneous amplitude measurements are made on a noise source, it will be found that most of measurements fall within a range around a certain value, the average value, and only a few have higher or lower values.

Some representative values for the Gaussian amplitude density distribution from Eq. (1-5) and amplitude distribution from Eq. (1-6) are given in table 1-1. The two functions are plotted (Fig. 1-4). From the table, if the noise rms voltage (σ) is, say, 150 mV, then the amplitude distribution function, *P(v)*, shows that 84% of all noise voltage values will be less than 1σ (150 mV) and 99.9% of all values will be less than 3σ (450 mV). Also, the probability of the voltage falling between –150 mV (–1σ) and +150 mV (+1σ) is 84.13% – 15.87% = 68.26% and between –450 mV (–3σ) and +450 mV (+3σ) is 99.865% – 1.350% = 98.52%. It should be noted that we assume the noise has no dc component (this is explicitly true when the noise is capacitor coupled). Thus, the average value of the noise voltage over any practical time period is zero.

Table 1-1 ~ Gaussian amplitude distribution functions
Note: p(v) is normalized to the rms value (σ)

v	**p(v)**	**P(v)**
-5σ	0.000 001 487/σ	0.000 000 287
-4σ	0.000 133 8/σ	0.000 031 67
-3σ	0.004 432/σ	0.001 350
-2σ	0.053 99/σ	0.022 75
-1σ	0.241 97/σ	0.158 65
0σ	0.398 94/σ	0.500 00
+1σ	0.241 97/σ	0.841 34
+2σ	0.053 99/σ	0.977 25
+3σ	0.004 432/σ	0.998 650
+4σ	0.000 133 8/σ	0.999 968 33
+5σ	0.000 001 487/σ	0.999 999 713

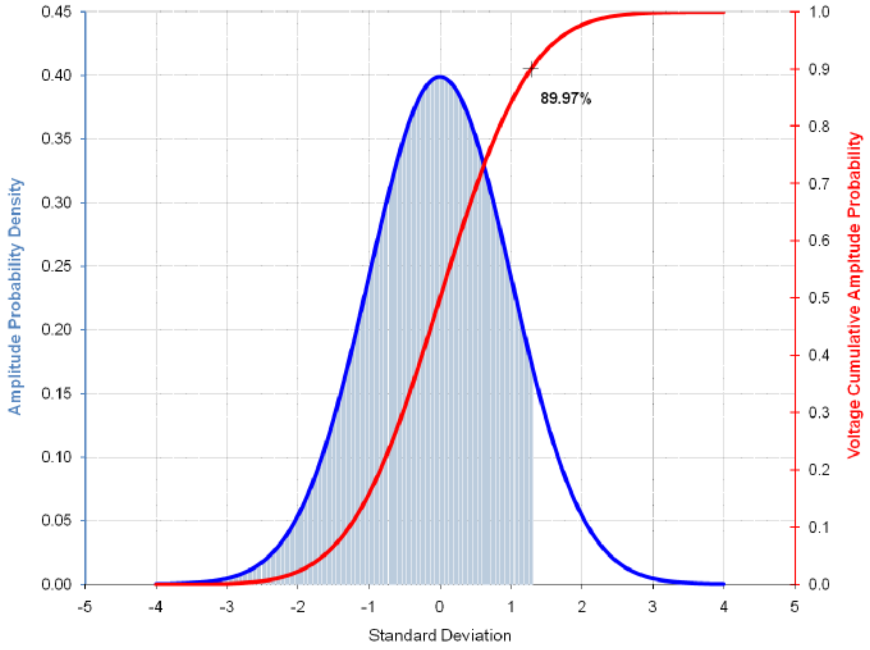
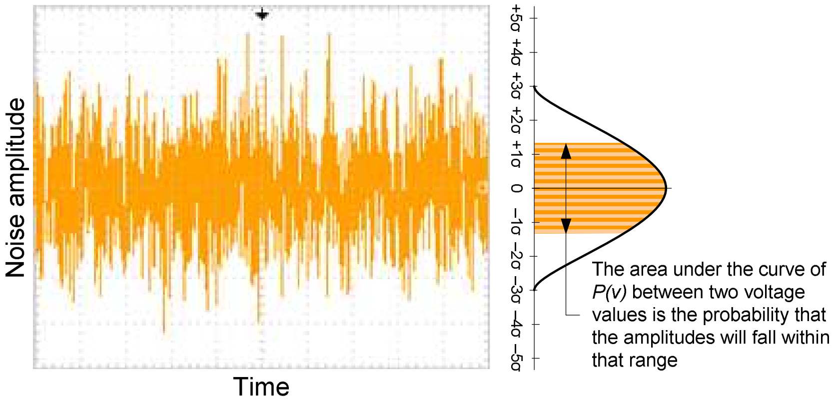


Fig. 1-4 ~ Gaussian amplitude probability density function (left scale and blue curve) and voltage cumulative amplitude distribution function (right scale and red curve) plotted together. The cross (+) indicates an example where the cumulative probability (red curve) and area under the amplitude density curve (blue shaded area) are 89.97%.

If we capture an oscilloscope display of Gaussian noise, it can be compared to the normal distribution curve (Fig. 1-5). Examination will show there are only a few instances when the noise voltage exceeds some high threshold, say 4σ, and many more instances when it exceeds a much lower threshold, say 1σ. In Part II we will discuss additional noise concepts including spectrum and temperature.



The area under the curve of *P(v)* between two voltage values is the probability that the amplitudes will fall within that range

Fig. 1-5 ~ Oscilloscope display of the output from a random noise generator. The generator has a bandwidth of 20 MHz and its output was set to 150 mV_{rms} (σ = 150 mV_{rms}). The scope vertical gain was set to 200 mV/division and the time base to 2.5 µs/division. A few noise peaks exceeded 700 mV (±4.7σ)

1-4. References

[ITU-R P-372.9]	Recommendation ITU-R P-372.9, Radio Noise, International Telecommunications Union, Radio Communications Sector, 2007

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