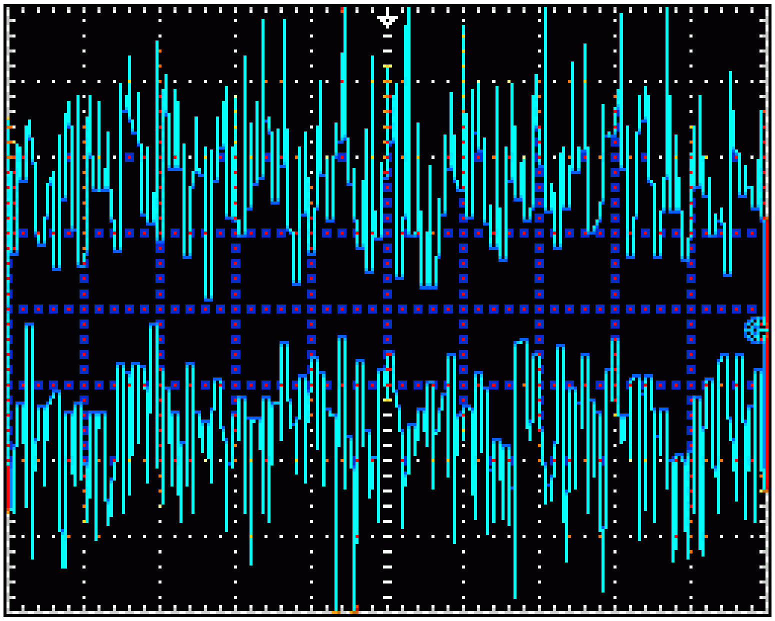
Noise Tutorial
Part II ~ Additional Noise Concepts



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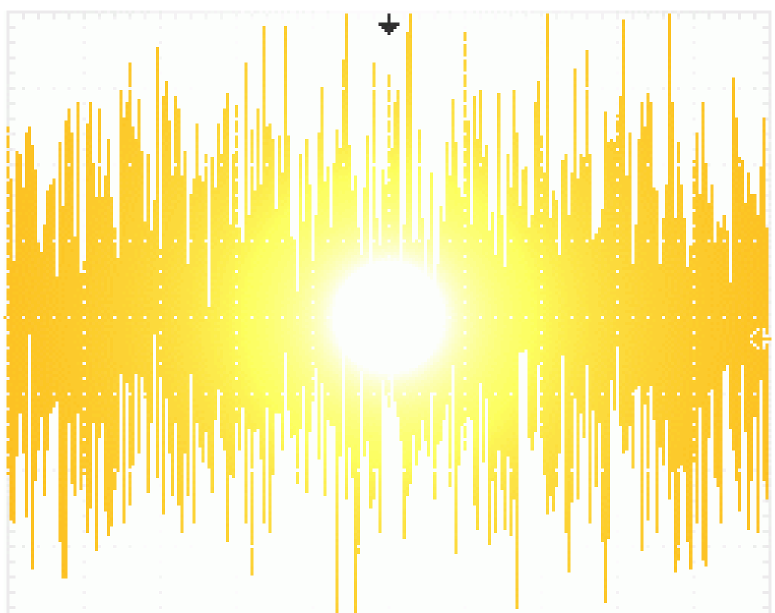

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Noise Tutorial II ~ Additional Noise Concepts

Abstract: With the exception of some solar radio bursts, the extraterrestrial emissions received on Earth's surface are very weak. Noise places a limit on the minimum detection capabilities of a radio telescope and may mask or corrupt these weak emissions. An understanding of noise and its measurement will help observers minimize its effects. This paper is a tutorial and includes six parts.

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Part II ~ Additional Noise Concepts

2-1. Noise spectrum

As mentioned in Part I, the spectrum of random noise contains no periodic frequency components and is a continuous function of frequency. The *spectral intensity* of the noise describes its frequency content and is given in units of voltage squared per unit bandwidth. When this is divided by the resistance across which the voltage is measured, it is equal to the power dissipated in the resistance per unit bandwidth and is called *power spectral density* (PSD, also called spectral power density). The most common unit for SPD is watts/hertz (W/Hz). Noise spectrum often is measured as a voltage, rather than power, in a given bandwidth. In this case, we define the *voltage spectrum* of the noise as numerically equal to the square root of the spectral intensity in units of voltage per square root of bandwidth. Since this is an unwieldy unit, it frequently is shortened to just volts per hertz (but it really is V/ $\sqrt{Hz}$ or volts per root hertz).

Noise that has a constant power spectral density over the range of frequencies in which we are interested is called *white noise*. The term white noise is analogous to white light, which is a superposition of all visible spectral components. White noise does not mean it contains equal amplitudes at all frequencies because the total noise power would then be infinite. Instead, it means the spectrum is flat over the range of interest, for example, the audio frequency range or receiver intermediate frequency bandwidth.

2-2. Noise bandwidth

Noise passes through various filters in any receiving system. The antenna, transmission lines, receiver input circuits, amplifiers, detectors and output circuits all have bandwidth limits. Filtering generally makes non-Gaussian noise more Gaussian. The concept of *noise bandwidth* applies to filters.

When white noise is presented to a receiver, some of the noise power will be rejected by the input filter and dissipated as heat or reflected back to the source. The amount of noise allowed to pass is determined by the filter's noise bandwidth. The noise bandwidth is the bandwidth of an ideal filter with a rectangular response that passes the same noise power as the real filter. The noise bandwidth is obtained by finding the area under the actual curve of the filter response with respect to frequency and converting it to an equivalent flat-top rectangular pass band with the same area. The height of the equivalent rectangular pass band is made equal to the maximum response of the actual filter curve.

If the mathematical function describing the actual filter response curve is known, the area can be found by integration from $-\infty$ to $+\infty$ in frequency ($-f_s/2$ to $+f_s/2$ for a digital filter sampled at f_s rate). Alternately, the area may be found graphically by plotting the response function or results of measurements. The response curve must be plotted in linear amplitude units and not logarithmic units such as dB and with linear frequency and not log frequency. For graphical analysis, plotting the frequency response to the –60 dB bandwidth values usually is sufficient.

For measured filter data, such as s-parameters, the measurements must be made with linear frequency and not log frequency and a sufficiently small frequency step size must be used. If the measurement results are in dB, they must be converted to linear ratios. For example, the s21 transmission parameters of a filter often are measured with a vector network analyzer as complex voltages but displayed or stored in dB (figure 2-1). For plotting purposes the measurements are converted to voltage magnitudes by

$$|s21| = 10^{\frac{s21(dB)}{20}}$$

Fig. 2-1 ~ Bandpass filter with –3 dB response from 20 to 90 MHz. The response is measured with a vector network analyzer setup for s-parameters and linear frequency sweep from 100 kHz to 180 MHz with 600 kHz step size (300 measurement points). The measurements are saved as a Touchstone *.s2p file for later processing. The noise bandwidth is the area under the filter response curve and is equivalent to the rectangular response shown with a thick red line.

The magnitude of s21 is a voltage so $|s21|^2$ is power normalized to 1 ohm impedance. The normalized power in each frequency increment of the filter response curve is

$$Area_{step} = |s21|^2 \cdot f_{step}$$

where

f_{step} frequency step size

$Area_{step}$ area for each frequency step

This is repeated across the entire frequency range and the results added,

$$Area_{Total} = \sum_{1}^{steps} \left(|s21|^2 \cdot f_{step} \right)$$

The bandwidth of a rectangular filter with equivalent noise bandwidth is then found by normalizing the total area to the maximum or peak value of the filter response, as in

$$B_n = \frac{Area_{Total}}{Peak|s21|^2} \text{ Hz}$$

Since the data in *.s2p files are ASCII text, they may be imported and analyzed in a spreadsheet program or in a program like Matlab; for example, see [Layne].

The noise bandwidth of a filter always is higher than its 3 dB signal bandwidth. The example of a bandpass filter was given above. A simple low-pass RC filter has a noise bandwidth 1.57 times its 3 dB bandwidth [Taub]. In a multi-stage tuned filter, the noise bandwidth factor is close to 1.05 to 1.1 giving a noise bandwidth only slightly more than its 3 dB bandwidth. Examples involving noise bandwidth are given in the following sections.

2-3. Noise temperature

Consider an ordinary resistor at temperature T (figure 2-1). As discussed in Part I, a resistor develops a thermal noise voltage. The corresponding noise power may be derived from the Rayleigh-Jeans and Planck radiation laws (for example, see [Nyquist]). The open-circuit noise voltage v_n across the resistor has a mean-square value in a frequency band B_n given by

$$v_n^2 = 4 \cdot k \cdot T \cdot R \cdot B_n \qquad (2\text{-}1)$$

where

- v_n open-circuit rms voltage (Vrms) (note that the function of time (t) is implied)
- k Boltzmann constant ($1.38 \cdot 10^{-23}$ J/K)
- T temperature (K)
- R resistance (ohms)
- B_n noise bandwidth (Hz)

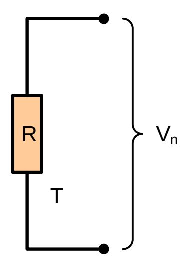


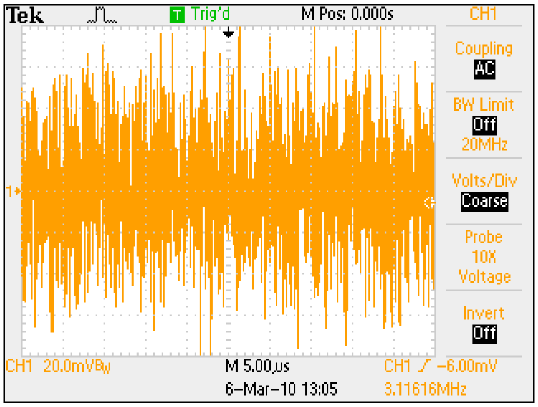
Fig. 2-1 ~ Resistor with resistance R at temperature T produces an open-circuit noise voltage V_n.

The Boltzmann constant k is a fundamental constant of physics occurring in nearly every statistical formulation of both classical and quantum physics. Its dimensions are energy (joules) per degree of absolute temperature (kelvin). The physical significance of k is that it indicates the amount of energy (heat) in a substance corresponding to the random thermal motions of its molecules.

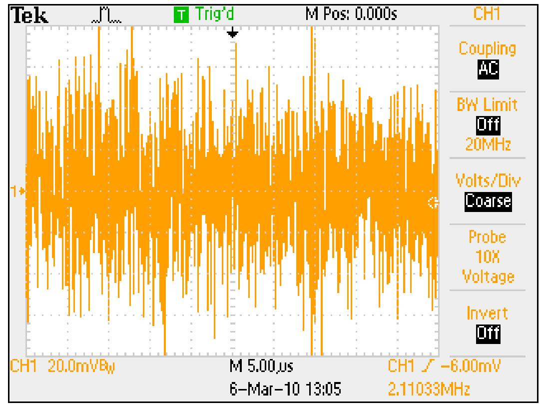
Taking the square root of both sides of Eq. (2-1),

$$v_n = \sqrt{4 \cdot k \cdot T \cdot R \cdot B_n} \text{ Vrms} \qquad (2\text{-}2)$$

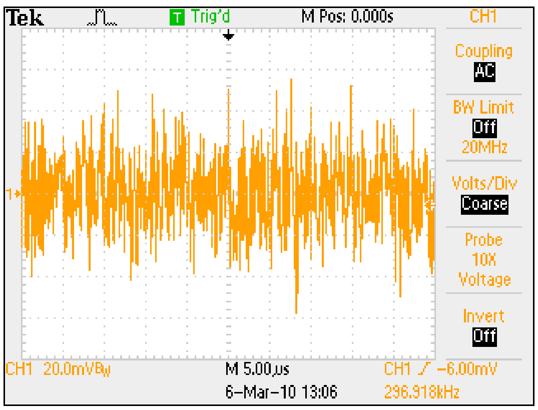
Since v_n^2 is proportional to power, Eq. (2-1) and (2-2) indicate that noise power is proportional to bandwidth and noise voltage is proportional to the square root of the bandwidth; for either case, the narrower the bandwidth the lower the noise power or voltage (figure 2-2).



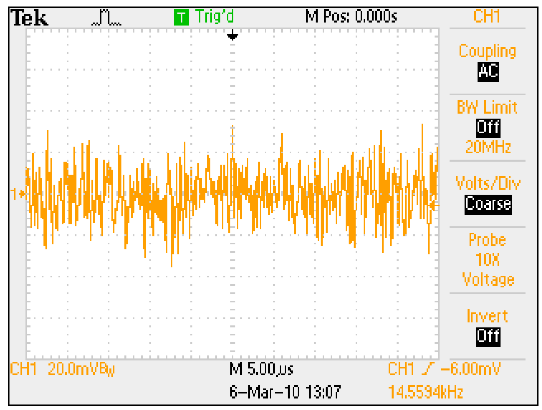
(a) 15 MHz



(b) 10.7 MHz



(c) 5 MHz



(d) 2.5 MHz

Fig. ~ 2-2 - Bandwidth limited noise voltages. The oscilloscope screenshots of voltage measurements on the output of various low-pass filters connected to a random noise generator whose frequency response is flat from 0 to 20 MHz and whose rms output voltage is set to the same value for all filters. All plots are shown at 20 mV/div vertical scale and 5 µs/div horizontal time scale giving full-scale spans of ±100 mV and 50 µs.

These relationships indicate that, to minimize the noise voltage (or noise power) in a receiving system, the bandwidth should be no higher than necessary. A system with excessively high bandwidth will detect too much noise and will not be as sensitive to weak signals as a system whose bandwidth is closer to the emissions of interest. On the other hand, the system bandwidth should not be too narrow or else too little of the desired emissions energy passes through the filter and it may not be detected.

A noisy resistor can be modeled as a series combination of a gaussian noise voltage generator and an ideal noiseless resistor (figure 2-3).

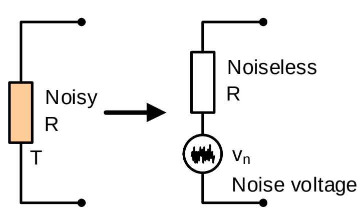


Fig. 2-3 ~ Noisy resistor model

When the resistor is placed under load, there will be some voltage drop across the source resistance R and some voltage drop across the load resistance R_L (figure 2-4). According to Kirchhoff's voltage law, the sum of the voltage drop across each resistor must equal the source voltage, or $V_n = V_R + V_L$.

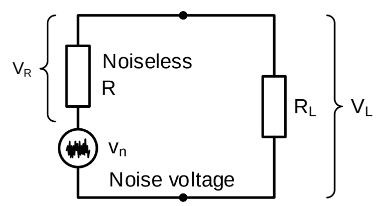


Fig. 2-4 ~ Noisy resistor model under load

Assuming both resistors are at temperature T, the maximum power delivered to an external load resistance R_L by the resistor R can be found by noting that maximum power transfer occurs when the load resistance equals the source resistance, in which case the voltage divides equally across the source and load, or

$$v_R = v_L = \frac{v_n}{2}$$
$$\frac{v_n}{2} = \frac{\sqrt{4 \cdot k \cdot T \cdot R \cdot B_n}}{2} = \frac{\sqrt{4 \cdot k \cdot T \cdot R_L \cdot B_n}}{2} \qquad (2\text{-}3)$$

2-4. Noise power

The average noise power N_{avg} delivered to the load is

$$N_{avg} = \frac{v_n^2}{4 \cdot R_L} = \frac{4 \cdot k \cdot T \cdot R_L \cdot B_n}{4 \cdot R_L} = k \cdot T \cdot B_n \qquad (2\text{-}4)$$

The units for the average power are $\left(\frac{J}{K}\right) \cdot (K) \cdot (Hz) = J \cdot Hz = \frac{J}{s} = W$

Eq. (2-4) indicates the average noise power is independent of the actual resistance values when the source and load are matched. The matching of source and load resistances in real receiving systems allows maximum transfer of the desired signal power from the source to the load, but it also allows the maximum transfer of noise power.

Example 2-1:

(a) The temperature used in many investigations is 290 K (17 °C). Find the open-circuit voltage of a 50 ohm resistor at this temperature in a bandwidth of 1 MHz

(b) Find the open-circuit voltage of the same resistor at the same temperature in a 1 Hz bandwidth

Solutions: Using Eq. (2-1)

(a) $v_n = \sqrt{4 \cdot k \cdot T \cdot R \cdot B_n} = \sqrt{4 \cdot 1.38 \cdot 10^{-23} \cdot 290 \cdot 50 \cdot 1 \cdot 10^{6}} = 0.895 \cdot 10^{-6}$ V_{rms} = 0.895 μV_{rms}

(b) $v_n = \sqrt{4 \cdot k \cdot T \cdot R \cdot B_n} = \sqrt{4 \cdot 1.38 \cdot 10^{-23} \cdot 290 \cdot 50 \cdot 1} = 0.895 \cdot 10^{-9}$ V_{rms} = 0.895 nV_{rms}

Comment: The ratio of bandwidths in (a) and (b) is 10^6, or 1,000,000. The square root of 10^6 is 10^3, so it should be no surprise that by decreasing the bandwidth by a factor of $1/10^6$ from 1 MHz to 1 Hz, the noise decreased by a factor of $1/10^3$ from 0.9 μV to 0.9 nV. These voltages are so small they are not measurable without considerable amplification.

Example 2-2:
Determine the average noise power delivered by a 50 ohm resistor at a temperature of 50,000 K to an ideal (noiseless) 50 ohm resistor in a 100 kHz bandwidth.

Solution:
Since the source and load are matched, we can use Eq. (2-4), or

$N_{avg} = k \cdot T \cdot B_n = 1.38 \cdot 10^{-23} \cdot 50{,}000 \cdot 100{,}000 = 6.94 \cdot 10^{-14}$ W = 0.0694 pW

Because the average thermal noise power is proportional to absolute temperature, it is common for noise power to be described in terms of noise temperature even though the noise may originate from something other than a hot resistance. For example, the noise could originate from the cosmic microwave background (CMB), a local interference source or a solar radio burst but we can refer to it in terms of its noise temperature.

In radio work a *reference temperature* T_0 = 290 K is used (see sidebar). Note the subscript for *T* is the number zero, or naught, and not the letter o. As a point of reference, the thermal noise power per hertz bandwidth (noise power density) available from a resistor at 290 K is

$N_{avg} = k \cdot T_0 \cdot B_n = 1.38 \cdot 10^{-23} \cdot 290 \cdot 1 = 4.0 \cdot 10^{-21}$ W/Hz =

$= 4.0 \cdot 10^{-9}$ pW/Hz

> The **reference temperature** 290 K is near Earth's average surface temperature (289 K), but that is only part of the reason for its choice. 290 K was chosen to make the value of kT easy to handle in computations [Friis44,45]. Some people incorrectly claim it is based on room temperature but 290 K is a cold room.
>
> Between the 1940s and 1950s the "standard" reference temperature varied from 288.44 to 300 K, depending on the author. The current value was adopted by the Institute of Radio Engineers (IRE) in 1952.

It often is convenient to calculate power levels in terms of deciBels with reference to 1 watt (dBW). For a temperature of 290 K the average power is

$N_{avg,dBW} = 10 \cdot \log_{10}\left(k \cdot T_0 \cdot B_n\right) = 10 \cdot \log_{10}\left(4.0 \cdot 10^{-21}\right) = -204$ dBW in a 1 Hz bandwidth.

For a 1 mW reference (dBm), add 30 dB to the previous calculation, or

$N_{avg,dBm} = -204 + 30 = -174$ dBm in a 1 Hz bandwidth.

These noise powers are written –204 dBW/Hz and –174 dBm/Hz. However, the practice of mixing logarithmic units (dBm) with linear units (Hz) is somewhat risky unless it is clearly understood that multiplication by the bandwidth does not yield the total power. The powers must first be converted back to linear units (W/Hz or mW/Hz) before the multiplication. The conversion is

$$N_{avg} = 10^{\frac{N_{dBm/Hz}}{10}} \text{ mW/Hz} \quad (2\text{-}5)$$

For example, the quantity –174 dBm/Hz converted to linear units of mW/Hz is

$$N_{avg} = 10^{\frac{N_{dBm/Hz}}{10}} = 10^{\frac{-174}{10}} = 4.0 \cdot 10^{-18} \text{ mW/Hz}$$

The total power in a 1 MHz bandwidth for the same noise temperature is

$$N_{Total} = 4.0 \cdot 10^{-18} \cdot B_n = 4.0 \cdot 10^{-18} \cdot 1 \cdot 10^{6} = 4.0 \cdot 10^{-12} \text{ mW in 1 MHz bandwidth}$$

and in decibels is

$$N_{Total} = 10 \cdot \log\left(4.0 \cdot 10^{-12}\right) = -114 \text{ dBm}$$

The total noise power calculation in dB can be written in a more general way as

$$N_{Total} = 10 \cdot \log\left(P_{avg} \cdot B_n\right) = N_{avg,dBm} + 10 \cdot \log\left(B_n\right) \text{ dBm} \quad (2\text{-}6)$$

It may be concluded that for a system at a noise temperature of 290 K, the noise floor is –174 dBm (or –204 dBW) in a 1 Hz bandwidth. Any noise-like emission below this power level, adjusted for bandwidth, cannot be distinguished from ordinary thermal noise.

2-5. Combinations of noisy resistors

We will now consider a number of noisy resistors and their combined effects. The illustration shows four noisy resistors, R_1 through R_4, connected in parallel with a hypothetical noiseless load resistor R_L (figure 2-5). We will calculate the noise voltage across R_L caused by the four noisy resistors. The concept can be easily extended to any number of noisy resistors.

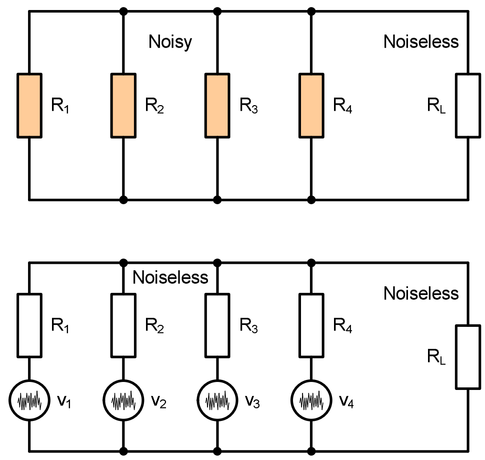


Fig. 2-5 ~ Combining noise sources. The noisy resistors (upper) are replaced by their equivalent noise models (lower).

Since noise voltages are uncorrelated (that is, the voltage at any instant is independent of the voltage at any other instant), we cannot simply add the individual voltages. Instead, we must calculate the noise power associated with each source and then add the powers together. To make the calculations we follow these steps:

1) Using Eq. (2-1) calculate the open-circuit noise voltage of each source based on its resistance and temperature
2) Short out all the noise voltage sources but one
3) Using ordinary circuit analysis, find the voltage across R_L due to the one active source
4) Calculate the power dissipated in R_L due to the one active source
5) Repeat steps 2) through 4) for each noise source in succession.
6) Add all the noise powers together on a linear basis to find the total noise power dissipated in R_L
7) Calculate the total noise voltage by solving

$$N_{Total} = \frac{v_{Total-n}^2}{R_L} \tag{2-7}$$

for $v_{Total-n}$, or

$$v_{Total-n} = \sqrt{N_{Total} \cdot R_L} \tag{2-8}$$

<u>Example 2-3</u>:

Find the noise voltage across an ideal load resistor R_L for the following resistor values and temperatures based on 100 kHz bandwidth:

R_L: 50 ohms; R_1: 100 ohms at 300 K; R_2: 1,000 ohms at 600 K; R_3: 2,000 ohms at 1200 K; R_4: 3,000 ohms at 2400 K

<u>Solution</u>:

Step 1: Referring to the circuit values, calculate the open circuit noise voltage for each resistor from Eq. (2-1)

$R_1 \rightarrow \quad v_1 = \sqrt{4 \cdot k \cdot T \cdot R \cdot B_n} = \sqrt{4 \cdot 1.38 \cdot 10^{-23} \cdot 300 \cdot 100 \cdot 100 \cdot 10^3} = 4.07 \cdot 10^{-7}\ V_{rms}$

$R_2 \rightarrow \quad v_2 = \sqrt{4 \cdot 1.38 \cdot 10^{-23} \cdot 600 \cdot 1000 \cdot 100 \cdot 10^3} = 1.82 \cdot 10^{-6}\ V_{rms}$

$R_3 \rightarrow \quad v_3 = \sqrt{4 \cdot 1.38 \cdot 10^{-23} \cdot 1200 \cdot 2000 \cdot 100 \cdot 10^3} = 3.64 \cdot 10^{-6}\ V_{rms}$

$R_4 \rightarrow \quad v_4 = \sqrt{4 \cdot 1.38 \cdot 10^{-23} \cdot 2400 \cdot 3000 \cdot 100 \cdot 10^3} = 6.30 \cdot 10^{-6}\ V_{rms}$

Step 2: Short out v_2 through v_4 (figure 2-9). The only active noise voltage source is v_1.

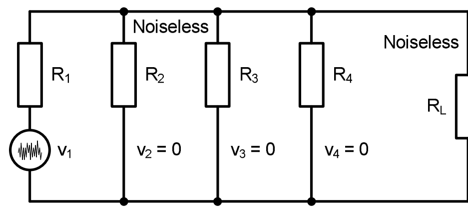


Fig. 2-9 – Example noise resistor network with v_2 through v_4 shorted out

Step 3: Solve for the voltage across R_L. The circuit can be reduced to a simple voltage divider circuit in which R_L is in parallel with R_2, R_3 and R_4 (figure 2-6). Using Kirchhoff's voltage law, the voltage V_{L-1} across R_L due to v_1 is (the $\|$ symbol indicates a calculation involving parallel resistors)

$$v_{L-1} = v_1 \cdot \left[\frac{R_1}{R_1 + (R_L \| R_2 \| R_3 \| R_4)} \right] = 4.07 \cdot 10^{-7} \cdot \left[\frac{100}{100 + 45.80} \right] = 2.79 \cdot 10^{-7}\ V_{rms}$$

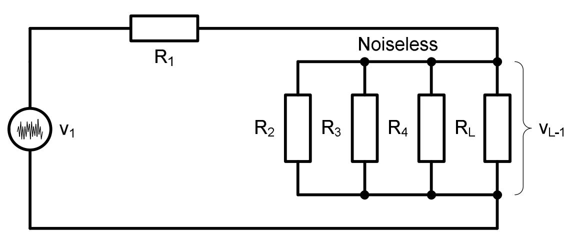


Fig. 2-6 – Equivalent voltage divider circuit. V_{L-1} is the noise voltage across the load resistor due to R_1

Step 4: Calculate the noise power dissipated in R_L due to the one active noise voltage source. For the first noise voltage source

$$N_{L-1} = \frac{v_{L-1}^2}{R_L} = \frac{\left(2.79 \cdot 10^{-7}\right)^2}{50} = 1.56 \cdot 10^{-15}\ W$$

Step 5: Repeat steps 2) through 4) for each noise source

5.2): v_2 is shorted out

5.3): $v_{L-2} = v_2 \cdot \left[\dfrac{R_2}{R_2 + (R_L \| R_1 \| R_3 \| R_4)} \right] = 1.82 \cdot 10^{-6} \cdot \left[\dfrac{1{,}000}{1000 + 32.43} \right] = 1.76 \cdot 10^{-6}\ \ \mathrm{V_{rms}}$

5.4): $N_{L-2} = \dfrac{v_{L-2}^2}{R_L} = \dfrac{\left(1.76 \cdot 10^{-6}\right)^2}{50} = 6.22 \cdot 10^{-14}\ \ \mathrm{W}$

5.2): v_3 is shorted out

5.3): $v_{L-3} = v_3 \cdot \left[\dfrac{R_3}{R_3 + (R_L \| R_1 \| R_2 \| R_4)} \right] = 3.64 \cdot 10^{-6} \cdot \left[\dfrac{2000}{2000 + 31.91} \right] = 3.58 \cdot 10^{-6}\ \ \mathrm{V_{rms}}$

5.4): $N_{L-3} = \dfrac{v_{L-3}^2}{R_L} = \dfrac{\left(3.58 \cdot 10^{-6}\right)^2}{50} = 2.57 \cdot 10^{-13}\ \ \mathrm{W}$

5.2): v_4 is shorted out

5.3): $v_{L-4} = v_4 \cdot \left[\dfrac{R_4}{R_4 + (R_L \| R_1 \| R_2 \| R_3)} \right] = 6.30 \cdot 10^{-6} \cdot \left[\dfrac{3000}{3000 + 31.75} \right] = 6.23 \cdot 10^{-6}\ \ \mathrm{V_{rms}}$

5.4): $N_{L-4} = \dfrac{v_{L-4}^2}{R_L} = \dfrac{\left(6.23 \cdot 10^{-6}\right)^2}{50} = 7.77 \cdot 10^{-13}\ \ \mathrm{W}$

Step 6: Add the noise powers to find the total noise power dissipated in R_L,

$$N_{Total} = N_{L-1} + N_{L-2} + N_{L-3} + N_{L-4} = 1.56 \cdot 10^{-15} + 6.22 \cdot 10^{-14} + 2.57 \cdot 10^{-13} + 7.77 \cdot 10^{-13} = 1.10 \cdot 10^{-12}\ \ \mathrm{W}$$

Step 7: Calculate the noise voltage across R_L,

$$v_{Total-4} = \sqrt{N_{Total} \cdot R_L} = \sqrt{1.10 \cdot 10^{12} \cdot 50} = 7.41 \cdot 10^{-6}\ \ \mathrm{V_{rms}}$$

Where m resistors are connected in series (figure 2-7), the total resistance R_T is simply the sum of the individual resistances, or

$$R_T = R_1 + R_2 + \ldots + R_m \qquad (2\text{-}9)$$

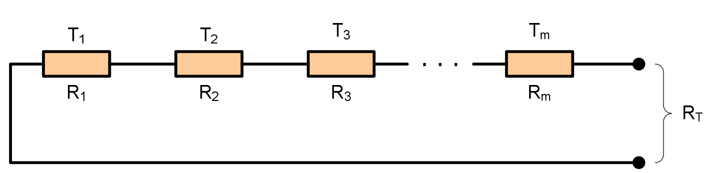


Fig. 2-7 – Series resistors

Again, the noise voltages are uncorrelated so we determine the mean-square noise voltages for the series combination. The total mean-square noise voltage is

$$v_{Total}^2 = v_1^2 + v_2^2 + ... + v_m^2 \tag{2-10}$$

If all resistors in series are the same temperature, that is, $T_1 = T_2 = T_3 \ldots = T_m = T$, it follows that

$$v_{Total}^2 = 4 \cdot k \cdot T \cdot B_n \cdot R_1 + 4 \cdot k \cdot T \cdot B_n \cdot R_2 + ... + 4 \cdot k \cdot T \cdot B_n \cdot R_m \tag{2-11}$$

or

$$v_{Total}^2 = 4 \cdot k \cdot T \cdot B_n \cdot (R_1 + R_2 + ... + R_m) = 4 \cdot k \cdot T \cdot B_n \cdot R_T \tag{2-12}$$

As expected, the available noise power N_{Total} is

$$N_{Total} = \frac{v_{Total}^2}{4 \cdot (R_1 + R_2 + ... + R_m)} = \frac{4 \cdot k \cdot T \cdot B_n \cdot (R_1 + R_2 + ... + R_m)}{4 \cdot (R_1 + R_2 + ... + R_m)} = k \cdot T \cdot B_n \tag{2-13}$$

If the resistors are at different temperatures, that is, $T_1 \neq T_2 \neq T_3 \ldots \neq T_m$,

$$v_{Total}^2 = 4 \cdot k \cdot T_1 \cdot B_n \cdot R_1 + 4 \cdot k \cdot T_2 \cdot B_n \cdot R_2 + ... + 4 \cdot k \cdot T_m \cdot B_n \cdot R_m \tag{2-14}$$

and the total available noise power is

$$N_{Total} = \frac{v_{Total}^2}{4 \cdot (R_1 + R_2 + ... + R_m)} = \frac{4 \cdot k \cdot T_1 \cdot B_n \cdot R_1 + 4 \cdot k \cdot T_2 \cdot B_n \cdot R_2 + ... + 4 \cdot k \cdot T_m \cdot B_n \cdot R_m}{4 \cdot (R_1 + R_2 + ... + R_m)} \tag{2-15}$$

or, equivalently,

$$N_{Total} = k \cdot B_n \cdot \left(\frac{T_1 \cdot R_1 + T_2 \cdot R_2 + ... + T_m \cdot R_m}{(R_1 + R_2 + ... + R_m)} \right) = \frac{k \cdot B_n}{R_T} \cdot (T_1 \cdot R_1 + T_2 \cdot R_2 + ... + T_m \cdot R_m) \tag{2-16}$$

We can now determine an equivalent temperature T_{equiv} for the series resistor combination, or

$$T_{equiv} = \frac{T_1 \cdot R_1 + T_2 \cdot R_2 + ... + T_m \cdot R_m}{(R_1 + R_2 + ... + R_m)} \tag{2-17}$$

The foregoing expression effectively weights each temperature according to the corresponding resitance with larger resistances having more weight. The individual contribution of each resistance to the total equivalent noise temperature, expressed as a fraction α, is

$$T_{equiv} = \alpha_1 \cdot T_1 + \alpha_2 \cdot T_2 + \ldots + \alpha_m \cdot T_m \tag{2-18}$$

where

$$\alpha_m = \frac{R_m}{\left(R_1 + R_2 + \ldots + R_m\right)} \tag{2-19}$$

Finally, if a matched noise source is connected to the series resistances, the total power available from that source would be absorbed by the resistances and each resistance would absorb a fraction of the available power according to

$$\frac{N_m}{N_{Total}} = \frac{R_m}{\left(R_1 + R_2 + \ldots + R_m\right)} = \alpha_m \tag{2-20}$$

<u>Example 2-4</u>:
For the series resistor combination (figure 2-8), determine the effective noise temperature, fractional contribution α of each resistor, and total available noise power under the following conditions. All resistances are in ohms and the bandwidth is 6 kHz:
a) $T_1 = T_2 = T_3 = T_4 = 300$ K
b) $T_1 = 300$ K, $T_2 = 600$ K, $T_3 = 1{,}200$ K, $T_4 = 2{,}400$ K

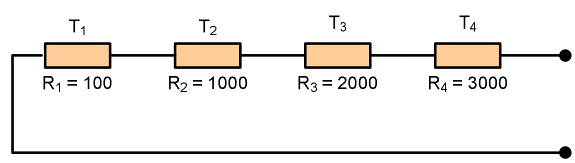


Fig. 2-8 – Example series resistor combination

<u>Solution</u>:
a) Since all resistors are the same temperature, $T_1 = T_2 = T_3 = T_4 = 300$ K, the equivalent noise temperature $T = 300$ K. To find the fractional contribution α_1 of R_1 use Eq. (2-19)

$$\alpha_1 = \frac{R_1}{\left(R_1 + R_2 + R_3 + R_4\right)} = \frac{100}{100 + 1000 + 2000 + 3000} = 0.0164$$

Similarly, for R_2, R_3, and R_4

$$\alpha_2 = \frac{R_2}{\left(R_1 + R_2 + R_3 + R_4\right)} = \frac{1000}{100 + 1000 + 2000 + 3000} = 0.1639$$

$$\alpha_3 = \frac{R_3}{\left(R_1 + R_2 + R_3 + R_4\right)} = \frac{2000}{100 + 1000 + 2000 + 3000} = 0.3279$$

$$\alpha_4 = \frac{R_4}{\left(R_1 + R_2 + R_3 + R_4\right)} = \frac{3000}{100 + 1000 + 2000 + 3000} = 0.4918$$

Note that $\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = 1.0000$. The total available power is found from Eq. (2-4)

$$N_{Total} = k \cdot T \cdot B_n = 1.38 \cdot 10^{-23} \cdot 300 \cdot 6 \cdot 10^3 = 2.48 \cdot 10^{-17} \text{ W}$$

b) For the case where each resistor is at a different temperature, the equivalent noise temperature is determined from Eq. (2-17)

$$T_{equiv} = \frac{T_1 \cdot R_1 + T_2 \cdot R_2 + \ldots + T_k \cdot R_m}{\left(R_1 + R_2 + \ldots + R_m\right)} = \frac{300 \cdot 100 + 600 \cdot 1000 + 1200 \cdot 2000 + 2400 \cdot 3000}{\left(100 + 1000 + 2000 + 3000\right)} = 1677 \text{ K}$$

The fractional contribution of each resistor is the same as found in a). The total available power is

$$N_{Total} = k \cdot T_{eq} \cdot B_n = 1.38 \cdot 10^{-23} \cdot 1677 \cdot 6 \cdot 10^3 = 1.39 \cdot 10^{-16} \text{ W}$$

Another way to find the total equivalent noise temperature is by noting that

$$N_{Total} = k \cdot T_{eq} \cdot B_n = k \cdot \left(\alpha_1 \cdot T_1 + \alpha_2 \cdot T_2 + T_3 + \alpha_3 \cdot T_4\right) \cdot B_n$$

Substituting the fractional contributions found in a),

$$N_{Total} = 1.38 \cdot 10^{-23} \cdot \left(0.0164 \cdot 100 + 0.1639 \cdot 600 + 0.3279 \cdot 1200 + 0.4918 \cdot 2400\right) \cdot 6 \cdot 10^3 = 1.39 \cdot 10^{-16} \text{ W}$$

as expected.

In Part III we derive the noise performance of attenuators and amplifiers.

2-6. References

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