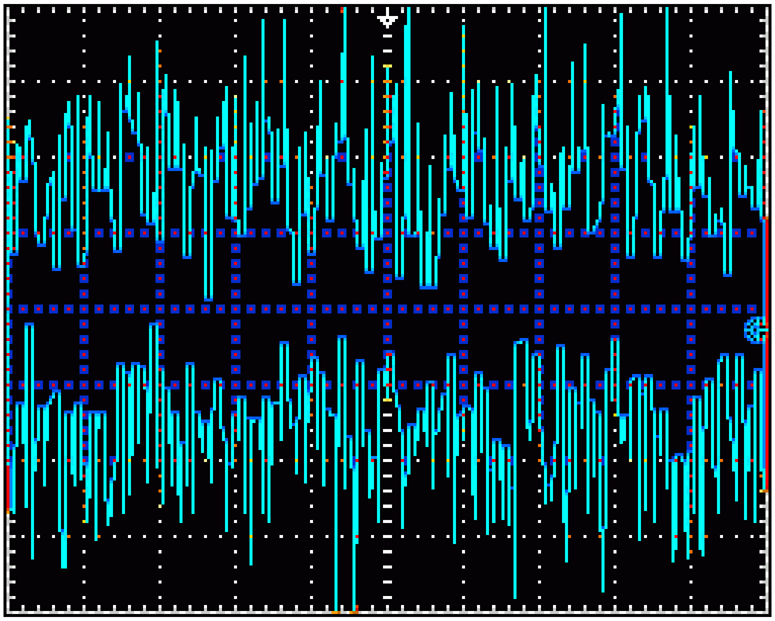
Noise Tutorial
Part V ~ Noise Factor Measurements



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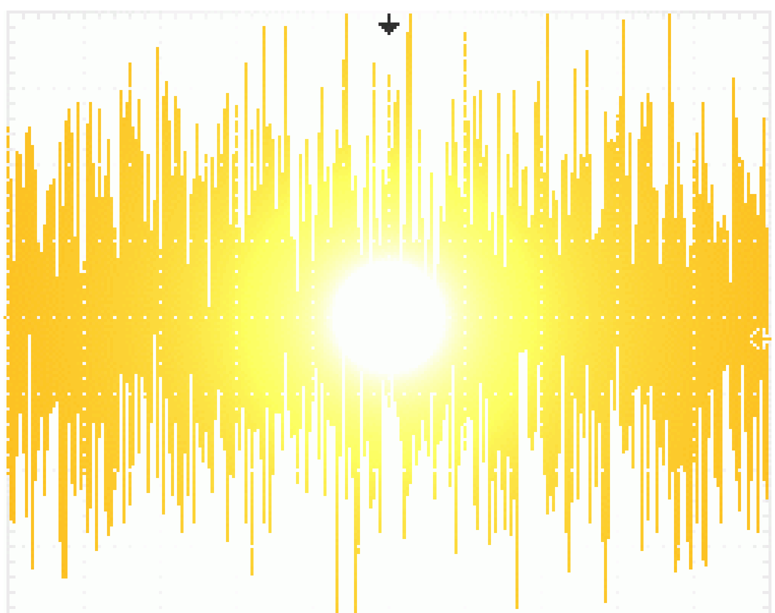

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Noise Tutorial V ~ Noise Factor Measurements

Abstract: With the exception of some solar radio bursts, the extraterrestrial emissions received on Earth's surface are very weak. Noise places a limit on the minimum detection capabilities of a radio telescope and may mask or corrupt these weak emissions. An understanding of noise and its measurement will help observers minimize its effects. This paper is a tutorial and includes six parts.

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Part V ~ Noise Factor Measurements

5-1. General considerations

Noise factor is an important measurement for amplifiers used in low noise applications such as radio telescopes and radar and other radio receivers designed to detect very low signal levels.
Noise factor is determined from noise power measurements. Noise power measurements may be obtained from a purpose-built noise figure meter (figure 5-1), a spectrum analyzer or even a modern vector network analyzer. Some receiver systems, for example, the Callisto solar radio spectrometer, can be used to measure the noise factor of external amplifiers. This part emphasizes using a spectrum analyzer.

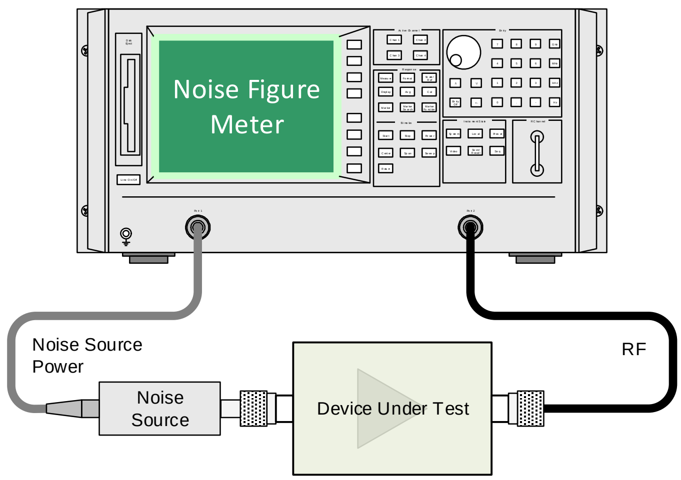

Fig. 5-1 ~ Noise figure meter with noise source and device under test

A calibrated noise source normally is used in noise power measurements. Commercial noise sources usually provide a flat noise power (or noise temperature) output over the bandwidth being measured. For example, if measurements are made on a wideband amplifier with a 500 MHz bandwidth in the frequency range 0.5 to 1.5 GHz, the noise source must cover this range. On the other hand, if a narrowband amplifier with 15 kHz bandwidth at 20 MHz is to be measured, the noise source only needs to cover 20 MHz ± 7.5 kHz. Most commercial noise sources have bandwidths above several GHz.

A noise source has two operational states, *cold* and *hot*. The cold state is an unpowered (off) state and the output is $k \cdot T_0 \cdot B_n$ thermal noise. The cold state noise power per Hz bandwidth at reference temperature T_0 is determined from the familiar calculation

$$P_{Cold} = P_0 = k \cdot T_0 \cdot B_n = 1.38 \cdot 10^{-23} \cdot 290 \cdot 1 = 4.002 \cdot 10^{-21} \text{ W/Hz}$$

In terms of noise power density expressed as dB with respect to 1 W

$$P_0 = 10 \cdot \log\left(4.002 \cdot 10^{-21}\right) = -203.98 \text{ dBW/Hz, rounded } -204 \text{ dBW/Hz}$$

and with respect to the more common 1 mW

$P_0 = -203.98 + 30 = -173.98$ dBm/Hz, rounded -174 dBm/Hz

The noise source hot state is the powered (on) state and it provides a known amount of noise in excess of the cold state noise. Common noise sources use a powering voltage of 28 Vdc (figure 5-2). The excess noise is expressed as an *Excess Noise Ratio*, or ENR, and is related to the noise power or noise temperature above the cold state noise by

$$ENR = \frac{T_{Hot} - T_{Cold}}{T_0} \tag{5-1}$$

where

T_{Hot} noise temperature when the noise source is in the hot state (powered, on)

T_{Cold} noise temperature when the noise source is in the cold state (unpowered, off)

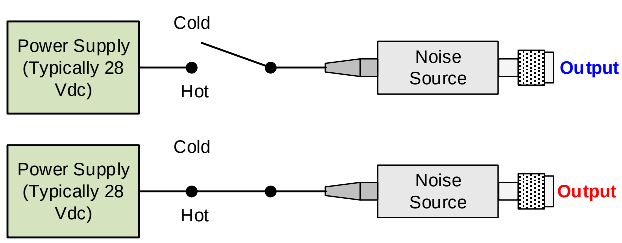

Fig. 5-2 ~ Noise source switching between cold (off) and hot (on) states

ENR normally is given as a logarithmic ratio in dB, or

$$ENR_{dB} = 10 \cdot \log\left(\frac{T_{Hot} - T_{Cold}}{T_0}\right) \tag{5-2}$$

For ordinary measurements $T_{Cold} = T_0$ but if the noise source is not at T_0, then Eq. (5-1) or (5-2) accounts for the difference. An undefined situation occurs when $T_{Hot} = T_{Cold}$ in which case $ENR_{dB} = 10 \cdot \log(0) = -\infty$ dB; therefore, in all practical measurements, $T_{Hot} > T_{Cold}$. If $T_{Hot} = 2 \cdot T_{Cold}$, then

$$ENR_{dB} = 10 \cdot \log\left(\frac{T_{Hot} - T_{Cold}}{T_0}\right) = 10 \cdot \log\left(\frac{2 \cdot T_{Cold} - T_{Cold}}{T_{Cold}}\right) = 10 \cdot \log(1) = 0\text{ dB}$$

Eq. (5-2) can be rewritten for the most common situation where $T_{Cold} = T_0$, or

$$ENR_{dB} = 10 \cdot \log\left(\frac{T_{Hot} - T_0}{T_0}\right) = 10 \cdot \log\left(\frac{T_{Hot}}{T_0} - 1\right) \tag{5-3}$$

It is seen that ENR is not simply the noise power above the quantity $k \cdot T_0 \cdot B_n$ or the noise temperature above T_0. Even when the noise source is off, it has a noise temperature T_0. The hot (on) state noise temperature may be determined in terms of the ENR_{dB} by solving Eq. (5-3) for T_{Hot}, or

$$T_{Hot} = T_0 \cdot 10^{\left(\frac{ENR_{dB}}{10}\right)} + T_0 = T_0 \cdot \left(10^{\left(\frac{ENR_{dB}}{10}\right)} + 1\right) \tag{5-4}$$

The most common excess noise ratios for commercial noise sources are 5, 6 and 15 dB but much higher ENRs are available. For example, the Renz RQ6 noise source is especially powerful with an ENR of 55 dB up to 3 GHz. Noise sources with 5 dB and 15 dB ENR have hot temperatures of

For ENR = 15 dB, $T_{Hot} = T_0 \cdot 10^{\left(\frac{ENR_{dB}}{10}\right)} + T_0 = 290 \cdot 10^{1.5} + 290 = 9460.6 \text{ K}$

For ENR = 5 dB, $T_{Hot} = 290 \cdot 10^{0.5} + 290 = 1207.1 \text{ K}$

The hot powers of these noise sources can be calculated by noting that the hot/cold powers are proportional to the hot/cold temperatures. Therefore,

$$ENR = \frac{P_{Hot} - P_0}{P_0} = \frac{P_{Hot}}{P_0} - 1 \tag{5-5}$$

Solving for P_{Hot} gives

$$P_{Hot} = P_0 \cdot (ENR + 1) \tag{5-6}$$

Equivalently,

$$P_{Hot} = P_0 \cdot \left(10^{\left(\frac{ENR_{dB}}{10}\right)} + 1\right) \tag{5-7}$$

For ENR = 15 dB,

$$P_{Hot} = P_0 \cdot \left(10^{\left(\frac{ENR_{dB}}{10}\right)} + 1\right) = 4.002 \cdot 10^{-21} \cdot \left(10^{\left(\frac{15}{10}\right)} + 1\right) = 1.306 \cdot 10^{-19} \text{ W/Hz}$$

The hot power in dBm/Hz is

$$P_{Hot} = 10 \cdot \log\left(1.306 \cdot 10^{-19}\right) + 30 = -158.84 \text{ dBm/Hz}$$

Similarly, for ENR = 5 dB,

$$P_{Hot} = 4.002 \cdot 10^{-21} \cdot \left(10^{\left(\frac{5}{10}\right)} + 1\right) = 1.666 \cdot 10^{-20} \text{ W/Hz}$$

and

$$P_{Hot} = 10 \cdot \log\left(1.666 \cdot 10^{-20}\right) + 30 = -167.78 \text{ dBm/Hz}$$

As mentioned in Part I, dBW/Hz and dBm/Hz are used for convenience in discussion and are not real units. One simply cannot multiply the noise powers in dBW/Hz or dBm/Hz by the bandwidth to determine the total noise power in a wider bandwidth. Instead, the powers must be converted to linear units (W/Hz or mW/Hz) before the multiplication and then re-converted back to decibel values. Alternately, the bandwidth can be converted to dB and then added to the noise power in dBW/Hz or dBm/Hz, as in

$$P_{Hot-dBm} = P_{Hot-dBm/Hz} + 10 \cdot \log\left(B_n\right) \tag{5-8}$$

Example 5-1: Find the noise power in milliwatts and dBm available from a 15 dB ENR noise source in the frequency range 250 to 750 MHz. The noise source output is flat over the frequency range 10 MHz to 10 GHz.

Solution: The noise power from this noise source was previously calculated as –158.84 dBm/Hz. The bandwidth is B_n = 750 – 250 = 500 MHz. Using the first method above, this value is converted to linear units, multiplied by the bandwidth in Hz and then converted back to dBm, or

$$P_{Hot,500MHz} = 10^{\left(\frac{P_{Hot,dBm/Hz}}{10}\right)} \cdot B_n = 10^{\left(\frac{-158.84}{10}\right)} \cdot 500 \cdot 10^6 = 1.306 \cdot 10^{-16} \cdot 500 \cdot 10^6 = 6.53 \cdot 10^{-8} \text{ mW}$$

In dBm, $P_{Hot,500MHz,dBm} = 10 \cdot \log\left(6.53 \cdot 10^{-8}\right) = -71.85$ dBm

Alternately, the bandwidth can be converted to dB and then added to the noise source power, or

$$P_{Hot,500MHz,\text{dBm}} = P_{Hot-dBm/Hz} + 10 \cdot \log\left(B_n\right) = -158.84 + 10\log\left(500 \cdot 10^6\right) = -158.84 + 86.99 = -71.85 \text{ dBm}$$

In milliwatts, $P_{Hot,500MHz} = 10^{\left(\frac{P_{Hot,500MHz,,dBm/Hz}}{10}\right)} = 10^{\left(\frac{-71.85}{10}\right)} = 6.53 \cdot 10^{-8}$ mW

If necessary, the ENR of a noise source may be reduced with an external attenuator. The calculation is the same as shown previously for an attenuator or transmission line, or

$$T_{Hot,\mathrm{A}} = T_{Hot} \cdot L_A + (1 - L_A) \cdot T_A \tag{5-9}$$

where

$T_{Hot,A}$ Hot state temperature of the noise source with the attenuator on its output

L_A Attenuator loss as a linear ratio of output power to input power

T_A Temperature of the attenuator or transmission line, usually T_0

The attenuated ENR is then calculated as before,

$$ENR_{dB,A} = 10 \cdot \log\left(\frac{T_{Hot,A} - T_{Cold}}{T_0}\right) = 10 \cdot \log\left(\frac{T_{Hot,A} - T_0}{T_0}\right) = 10 \cdot \log\left(\frac{T_{Hot,A}}{T_0} - 1\right)$$

Using a 15 dB ENR noise source with a 10 dB attenuator (0.1 linear power ratio), the new hot state temperature is

$$T_{Hot,\mathrm{A}} = T_{Hot} \cdot L_A + (1 - L_A) \cdot T_A = 9460.6 \cdot 0.1 + (1 - 0.1) \cdot 290 = 1207.1\ \mathrm{K}$$

and the new ENR is

$$ENR_{dB,A} = 10 \cdot \log\left(\frac{T_{Hot,A} - T_{Cold}}{T_0}\right) = 10 \cdot \log\left(\frac{1207.1 - 290}{290}\right) = 5\ \mathrm{dB}$$

In this example, there would have been no significant error in subtracting the attenuator value from the ENR (both in dB). Simple subtraction (in dB) is accurate for most practical situations involving typical noise sources and attenuator values. It should be noted that an attenuator on the output of a noise source can reduce impedance mismatch error, but error in the attenuation itself directly affects the ENR used in the noise factor calculations. For example, a +0.5 dB error in the attenuator value will cause a –0.5 dB error in the ENR (a 10 dB attenuator actually is 10.5 dB and a 5.0 dB ENR noise source actually will be 4.5 dB). It is for this reason that attenuators need to be accurately measured or precision attenuators be used with noise sources. There are many sources of error and uncertainty (see sidebar).

A very high value attenuator connected between a noise source and device simply provides a noise source with equal hot and cold temperatures and an undefined ENR as previously discussed. For example, if

Measurement uncertainty and mismatch error. All measurements are uncertain to some extent, and there are many subtle details that are important in accurate noise measurements. Uncertainties are especially important in measurements of low noise factors. For example, measurement of 0.5 dB noise factor can easily have more than 0.5 dB uncertainty when taking into account connectors, cables and equipment calibration. Also, an impedance mismatch between the noise source and device causes some noise power to be reflected back and unavailable for measurement. Measurement uncertainties and mismatch errors are dealt with in [Dunsmore].

a 60 dB attenuator at 290 K is applied to a noise source with ENR = 15 dB, calculation to 5 decimal places gives

$$T_{Hot,\mathrm{A}} = T_{Hot} \cdot L_A + (1 - L_A) \cdot T_A = 9460.60521 \cdot 0.000001 + (1 - 0.000001) \cdot 290 = 290.00917 \text{ K}$$

and

$$T_{Cold,\mathrm{A}} = T_{Cold} \cdot L_A + (1 - L_A) \cdot T_A = 290 \cdot 0.000001 + (1 - 0.000001) \cdot 290 = 290.00000 \text{ K}$$

5-2. Noise factor measurements with Y-factor method

One of several methods used to measure noise factor is called the Y-factor method. It is described in detail in [Agilent 57-2] and more briefly below. With this method, a pair of hot/cold measurements is taken and noise factor is then calculated from one of the following equations

$$NF = \frac{ENR}{Y - 1}$$
$$NF_{dB} = 10 \cdot \log\left(\frac{ENR}{Y - 1}\right) \tag{5-10}$$
$$NF_{dB} = 10 \cdot \log\left(\frac{10^{\frac{ENR_{dB}}{10}}}{Y - 1}\right) = ENR_{dB} - 10 \cdot \log(Y - 1)$$

where

$$Y = \frac{P_{Hot}}{P_{Cold}} \tag{5-11}$$

and

P_{Hot} noise power measured at the output of the device for the hot state, in suitable linear units
P_{Cold} noise power measured at the output of the device for the cold state, in same units as P_{Hot}

The noise powers may be measured many ways but a spectrum analyzer is described in detail in the next section. If the hot and cold noise powers are read from a spectrum analyzer in dBm,

$$Y_{dB} = P_{Hot,dBm} - P_{Cold,dBm} \tag{5-12}$$

and

$$NF_{dB} = 10 \cdot \log\left(\frac{10^{\frac{ENR_{dB}}{10}}}{10^{\frac{Y_{dB}}{10}} - 1}\right) = ENR_{dB} - 10 \cdot \log\left(10^{\frac{Y_{dB}}{10}} - 1\right) \tag{5-13}$$

Example 5-2: A noise source with ENR_{dB} = 5.32 dB is used to measure an amplifier with the following results: P_{Hot} = -118.0 dBm and P_{Cold} = -121.9 dBm. Find the noise factor.

Solution:
From Eq. (5-11), Y_{dB} = –118.0 dBm – (–121.9 dBm) = 3.9 dB
and from Eq. (5-12), $NF_{dB} = ENR_{dB} - 10 \cdot \log\left(10^{\frac{Y_{dB}}{10}} - 1\right) = 5.32 - 10 \cdot \log\left(10^{\frac{3.9}{10}} - 1\right) = 3.7\ \text{dB}$

Where the physical temperature of the noise source is $T_{Cold}^{'}$, Eq. (5-10) is modified

$$NF = \frac{ENR - Y\left(\frac{T_{Cold}^{'}}{T_0} - 1\right)}{Y - 1} \tag{5-14}$$

Note that Eq. (5-13) reduces to (5-10) when $T_{Cold}^{'} = T_0$.

The Y-factor method depends on the linearity of the devices in the measurement chain, so the noise source ENR should be as low as possible to avoid overdriving them. However, it should not be so low that the difference between the on and off noise powers is too small to be measured accurately.

There are no simple rules for matching noise source ENR to a device being measured. However, a general guideline for the Y-factor method is the ENR should be within about 10 dB of the device's noise factor. For example, a 5 dB ENR noise source may be used to measure noise factors up to about 15 dB, and a 15 dB ENR noise source should not be used to measure noise factors below approximately 5 dB or above 25 dB.

The Y-factor method measures the noise factor of the device on the basis of the noise source impedance. If the noise source does not match the device input impedance, the measurement will include mismatch error due to reflections from the device back to the noise source. It is the cold impedance that is important and the hot impedance less so. Noise sources with lower ENRs are built by adding an internal high-quality attenuator to a high ENR source, which improves both the cold and hot impedance match. Most low noise amplifier measurements will be at 50 ohms impedance.

Y-factor method procedure:

1. Connect the calibrated noise source to the device being measured using the highest quality and lowest loss coaxial cable possible or connect the noise source directly to the device
2. With no power applied to the noise source, measure the noise power at the device output (P_{Cold})

3. Apply power to the noise source and again measure the noise power at the device output (P_{Hot})
4. Calculate Y
5. Correct the noise source ENR for connecting cable loss (if any) between the noise source and the device and calculate NF
6. Be careful not to mix linear and logarithmic power ratios in the calculations

5-3. References

[Agilent 57-2] Noise Figure Measurement Accuracy – The Y-Factor Method, Application Note 57-2, Document No. 5952-3706E, Agilent Technologies, Inc. 2013

[Dunsmore] Dunsmore, J., Handbook of Microwave Component Measurements with Advanced VNA Techniques, John Wiley & Sons, 2012

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