

Ionosphere-Distance Calculations

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A. Great circle distance:

The total great circle arc angle Φ and distance d between two stations are found from the station coordinates (latitude and longitude).

$$\cos(\Phi) = \sin A \cdot \sin C + \cos A \cdot \cos C \cdot \cos \Delta L \quad (1)$$

where (all angles use the same units, all degrees or all radians)

Φ = Angle of the great circle arc from transmitter a and receiver c

A = latitude of station a (+ for northern hemisphere)

C = latitude of station c (+ for northern hemisphere)

ΔL = angular difference in longitude between stations a and c (magnitude > 0)

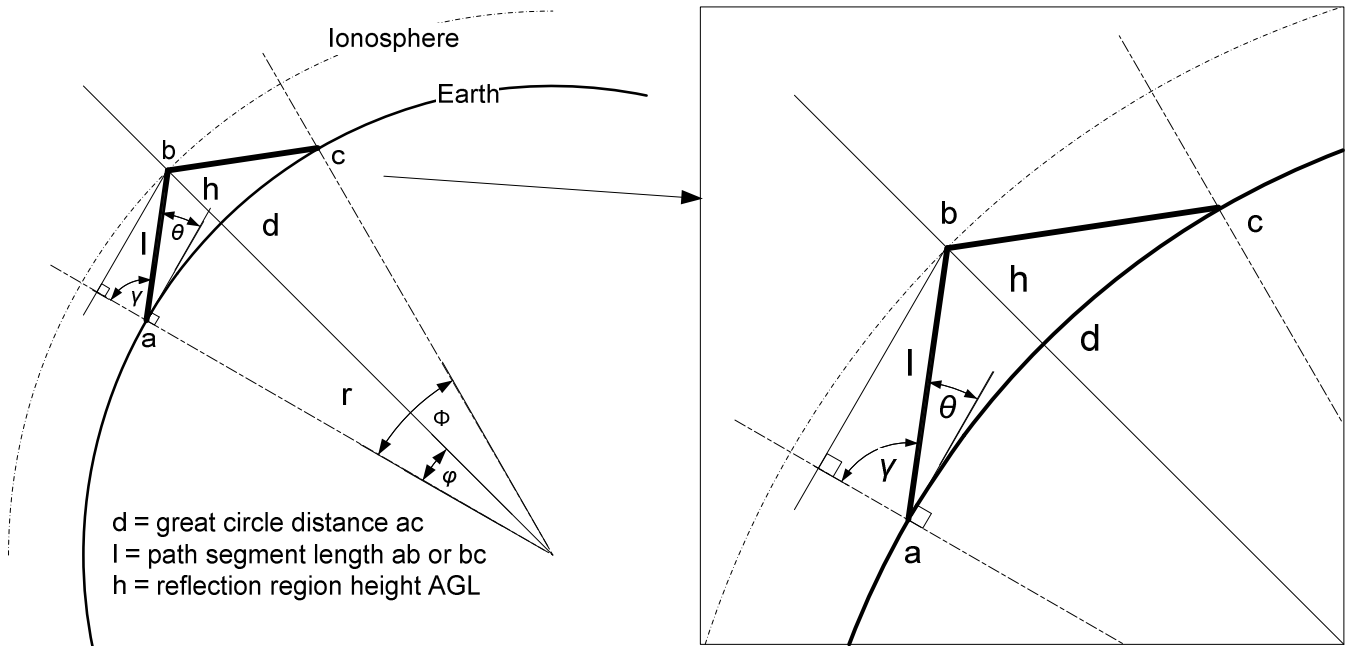
$$\Phi = \arccos(\sin A \cdot \sin C + \cos A \cdot \cos C \cdot \cos \Delta L) \quad (2)$$

$$d = r \cdot \Phi = 111.2 \cdot \Phi^\circ \text{ km} \quad (3.a)$$

$$d = r \cdot \Phi = 6370 \cdot \Phi^{rad} \text{ km} \quad (3.b)$$

For additional details on position, distance and bearing calculations, see [PDBC].

B. Radio path length:



A hop is defined as the radio path from the ground up to the reflection (refraction) region and back down to the ground. For a 1-hop circuit a-b-c, the total radio path length is $2 \cdot l$ and for an m-hop circuit the total path length is $2 \cdot m \cdot l$.

$$\varphi = \frac{\Phi}{2 \cdot m} \quad (4)$$

From Law of Cosines, the radio path segment length l is given by

$$l^2 = r^2 + (r+h)^2 - 2 \cdot r \cdot (r+h) \cdot \cos \varphi \quad (5a)$$

Equivalently,

$$l^2 = 2 \cdot r \cdot (r+h) \cdot (1 - \cos \varphi) + h^2 \quad (5b)$$

and

$$l = \sqrt{2 \cdot r \cdot (r+h) \cdot (1 - \cos \varphi) + h^2} \quad (5c)$$

where

φ = Angle of the great circle arc from transmitter a (or receiver c) and point on Earth below reflection (refraction) region b

l = path segment length from transmitter a (or receiver c) to reflection (refraction) region b (km)

r = Earth radius (6370 km)

h = height of reflection region above ground level (km)

C. Elevation angle:

By inspection

$$l \cdot \sin \gamma = (r+h) \cdot \sin \varphi \quad (6)$$

where

γ = angle between Earth radial to transmitter a (or receiver c) and radio path l . Note: $\gamma < \frac{\pi}{2}$ rad.

Therefore,

$$\sin \gamma = \frac{(r+h) \cdot \sin \varphi}{l} \quad (7)$$

and,

$$\sin \gamma = \frac{(r+h) \cdot \sin \varphi}{\sqrt{2 \cdot r \cdot (r+h) \cdot (1 - \cos \varphi) + h^2}} \quad (8)$$

Solving for γ gives

$$\gamma = \arcsin \left(\frac{(r+h) \cdot \sin \varphi}{\sqrt{2 \cdot r \cdot (r+h) \cdot (1 - \cos \varphi) + h^2}} \right) \quad (9)$$

By inspection, the elevation angle θ (radians) is

$$\theta = \frac{\pi}{2} - \gamma \quad (10)$$

and

$$\theta = \frac{\pi}{2} - \arcsin \left(\frac{(r+h) \cdot \sin \varphi}{\sqrt{2 \cdot r \cdot (r+h) \cdot (1 - \cos \varphi) + h^2}} \right) \quad (11)$$

Note: θ must be ≥ 0

References:

- [PDBC] Reeve, W., Position, Distance and Bearing Calculations, 2014,
http://www.reeve.com/Documents/Articles%20Papers/Reeve_PosDistBrngCalcs.pdf
- [SFD] Reeve, W., Sudden Frequency Deviations Due to Solar Flares, 2014,
http://www.reeve.com/Documents/Articles%20Papers/Reeve_SudFreqDev.pdf

Document Information

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