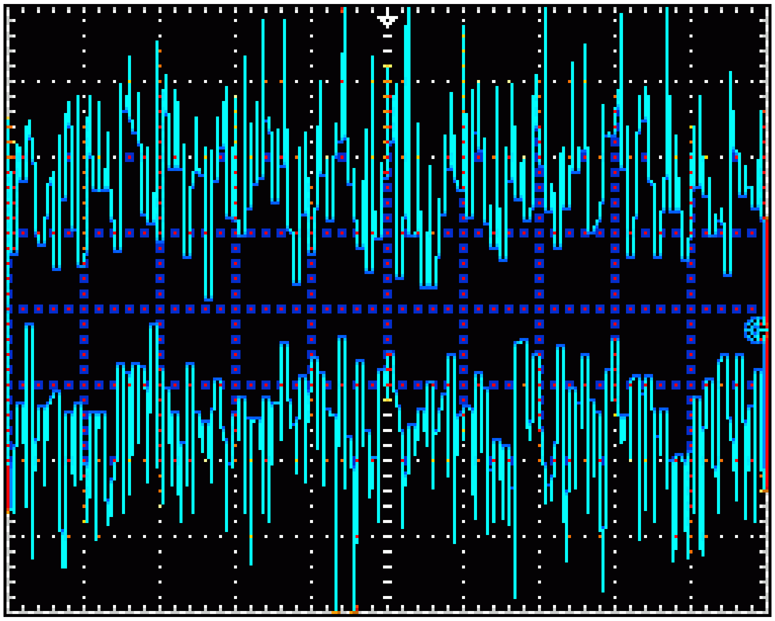
Noise Tutorial
Part IV ~ Noise Factor



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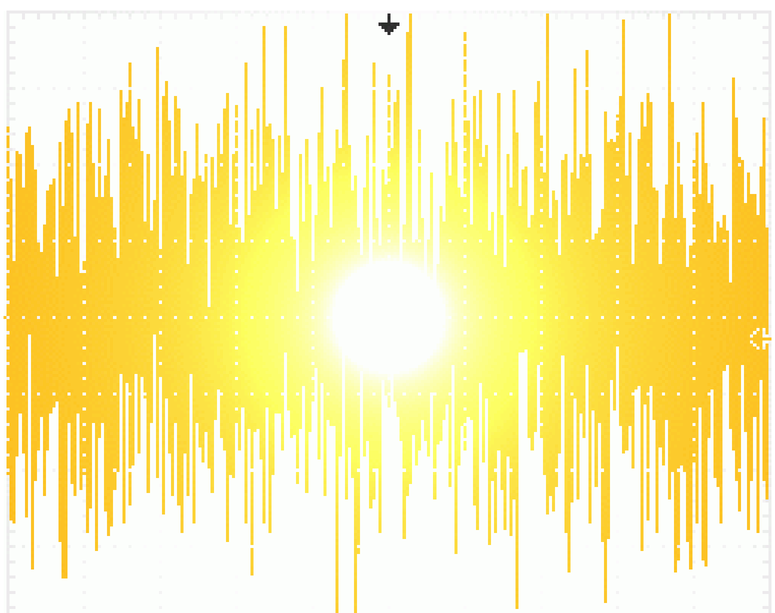

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Noise Tutorial IV ~ Noise Factor

Abstract: With the exception of some solar radio bursts, the extraterrestrial emissions received on Earth's surface are very weak. Noise places a limit on the minimum detection capabilities of a radio telescope and may mask or corrupt these weak emissions. An understanding of noise and its measurement will help observers minimize its effects. This paper is a tutorial and includes six parts.

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Part IV ~ Noise Factor

4-1. Noise factor and noise figure

Noise factor and *noise figure* indicates the noisiness of a radio frequency device by comparing it to a reference noise source. The *IEEE Standard Dictionary of Electrical and Electronics Terms* [IEEE100] does not distinguish between the terms *noise factor* and *noise figure* and defines them in the same way; however, noise factor commonly is expressed as a linear power ratio and noise figure as a logarithmic power ratio (dB). This discrepancy causes confusion and will be discussed in more detail later. In this paper only *noise factor* is used and it always is made clear if it is a linear or logarithmic power ratio.

One definition of noise factor NF relates the input signal-to-noise ratio (SNR) of a device to its output SNR, or

$$NF = \frac{S_{in}/N_{in}}{S_{out}/N_{out}} \tag{4-1}$$

where
S_{in} signal power at device input
S_{out} signal power at device output
N_{in} total noise power available at device input
N_{out} total noise power available at device output (including noise from the device itself)

All practical devices degrade the SNR, so the output SNR always will be lower (worse) than the input SNR (figure 4-1).

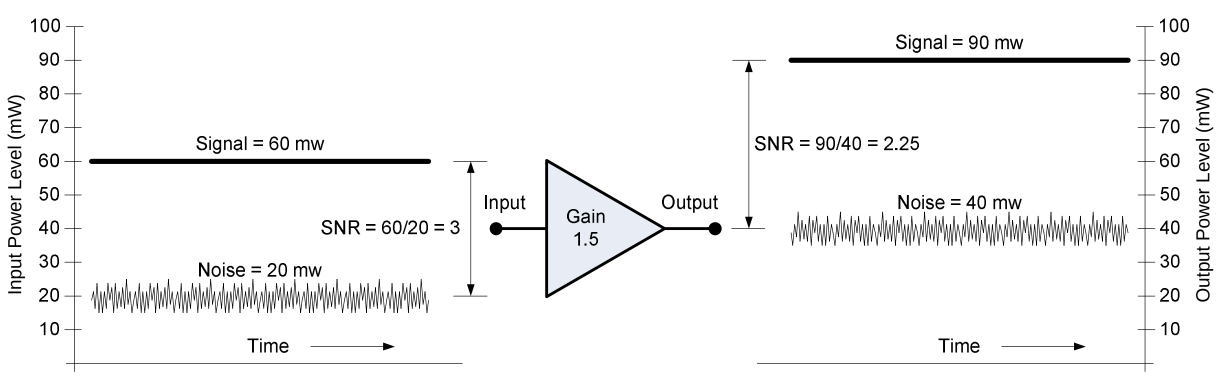


Fig. 4-1 ~ Amplifier signal and noise levels. Both the input signal and external noise are multiplied by the amplifier power gain; however, the amplifier adds even more noise at the output, thus lowering the output SNR.

Eq. (4-1) can be rearranged in terms of the device gain

$$NF = \frac{S_{in}/N_{in}}{G \cdot S_{in}/\left(G \cdot N_{in} + N_{Dev}\right)}$$
$$NF = \frac{S_{in}/N_{in}}{G \cdot S_{in}/G \cdot \left(N_{in} + N_{Dev-in}\right)} \qquad (4\text{-}2)$$

where

N_{Dev} additional noise due to the device itself, referred to its output ($= G \cdot N_{Dev-In}$)

$N_{Dev\text{-}in}$ additional noise due to the device itself, referred to its input

Eq. (4-2) provides equivalent expressions for the device noise. One uses a noisy device with its internal noise referred to the output (N_{Dev}), and the other uses the combination of a noiseless device with its internal noise referred to its input ($N_{Dev\text{-}in}$), where $N_{Dev} = G N_{Dev\text{-}in}$. Eq. (4-2) reduces to

$$NF = \frac{N_{in} + N_{Dev-in}}{N_{in}} = 1 + \frac{N_{Dev-in}}{N_{in}} \qquad (4\text{-}3)$$

Alternately,

$$NF = \frac{G \cdot N_{in} + N_{Dev}}{G \cdot N_{in}} = 1 + \frac{N_{Dev}}{G \cdot N_{in}} \qquad (4\text{-}4)$$

Eq. (4-3) can be rearranged

$$N_{Dev-in} = (NF - 1) \cdot N_{in} \qquad (4\text{-}5)$$

Similarly, for Eq. (4-4)

$$N_{Dev} = G \cdot (NF - 1) \cdot N_{in} \qquad (4\text{-}6)$$

Eq. (4-3) shows that the noise factor expresses the noisiness of a device relative to an input noise source N_{in}. Therefore, with a noise reference at the device input, the noise factor will represent a measure of how much noisier the device is than the reference. Using a standard noise reference temperature allows direct comparison of different devices. As pointed out in Part I, the reference temperature is defined as T_0 = 290 K. For this temperature, the reference noise power per unit bandwidth is

$$N_0 = k \cdot T_0 = 1.38 \cdot 10^{-23} \cdot 290 = 4.00 \cdot 10^{-21} \text{ W/Hz}$$

We can define an effective noise temperature T_{eff} such that the noise power produced by a device referred to its input is

$$N_{Dev-in} = k \cdot T_{eff} \cdot B_n \qquad (4\text{-}7)$$

The input noise power to the device from an external source at the reference noise temperature T_0 is

$$N_{in} = k \cdot T_0 \cdot B_n \tag{4-8}$$

Therefore, from Eq. (4-5),

$$k \cdot T_{eff} \cdot B_n = (NF - 1) \cdot k \cdot T_0 \cdot B_n \tag{4-9}$$

Cancelling terms gives

$$T_{eff} = (NF - 1) \cdot T_0 \tag{4-10}$$

Solving for the noise factor gives

$$NF = \frac{T_0 + T_{eff}}{T_0} = 1 + \frac{T_{eff}}{T_0} \tag{4-11}$$

We can use these results to also find the noise factor of a device that introduces loss, such as an attenuator or lossy transmission line. First, we substitute the standard noise reference temperature T_0 for T_A in Eq. (2-17) and then combine it with Eq. (4-10), or

$$T_{eff} = (NF_A - 1) \cdot T_0 = (L_A - 1) \cdot T_0 \tag{4-12}$$

where NF_A is the attenuator noise factor.

Simplifying,

$$(NF_A - 1) = (L_A - 1) \tag{4-13}$$

and solving for the noise factor of the attenuator,

$$NF_A = L_A \tag{4-14}$$

where L_A is defined as the ratio input power/output power and $L_A \geq 1$. This expression directly links the attenuator or transmission line loss to its noise factor. Examples are shown later.

It is important to note that noise factor is not an absolute measure of noise and that NF ≥ 1. For an ideal device (a device that does not contribute any noise) *NF* = 1.

As previously mentioned, the definitions of noise factor and noise figure are the same. Both can be expressed equivalently as linear or logarithmic power ratios. As a logarithmic ratio in dB

$$NF_{dB} = 10 \cdot \log(NF) \tag{4-15}$$

However, some important literature (for example, the often cited and excellent application note [Agilent 57-1]) makes a distinction and expresses noise factor as a linear ratio and noise figure as a logarithmic ratio, or

Noise factor = *NF*
Noise figure = $NF_{dB} = 10 \cdot \log(NF)$

To add even more confusion, some literature uses the letter *F* and shows noise factor in lower-case (*f*) and noise figure in upper-case (*F*) (for example, see [ITU-R P-372.9]). Unfortunately, lower-case (*f*) is used to represent frequency in electrical engineering literature. In this article we use *NF* to represent both noise factor and noise figure and always make it clear when a logarithmic ratio in dB is used.

Example 4-1:
Express the following noise factors in dB: (a) 1.0; (b) 1.15, (c) 3.0; (d) 10.0

Solution: Using Eq. (4-15)
(a) $NF_{dB} = 10 \cdot \log(1.0) = 0 \text{ dB}$
(b) $NF_{dB} = 10 \cdot \log(1.15) = 0.607 \text{ dB}$
(c) $NF_{dB} = 10 \cdot \log(3.0) = 4.771 \text{ dB}$
(d) $NF_{dB} = 10 \cdot \log(10.0) = 10 \text{ dB}$

Comment: In (a), the noise factor of 1.0 corresponds to an ideal noiseless device.

Example 4-2:
Find the noise factor and effective noise temperature of an amplifier for the conditions given (figure 4-2).

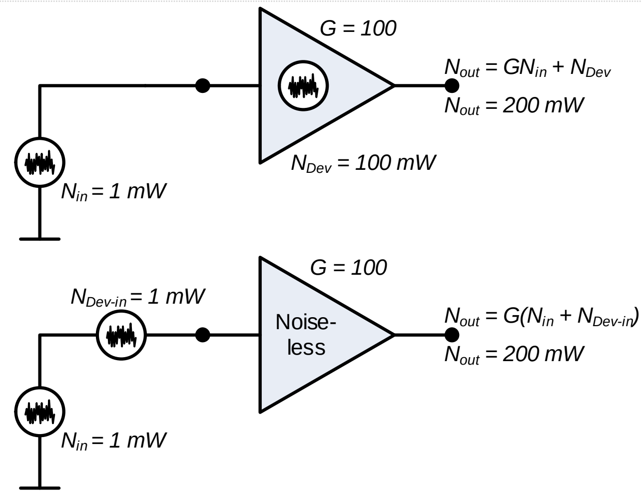


Fig. 4-2 ~ Noise factor of an amplifier. Two equivalent representations are shown, one with the internal noise referred to the output (upper) and the other with the internal noise referred to the input

Solution:

For the upper drawing use Eq. (4-4), or

$$NF = \frac{G \cdot N_{in} + N_{Dev}}{G \cdot N_{in}} = \frac{100 \cdot 1 + 100}{100 \cdot 1} = 2$$

For the lower drawing use Eq. (4-3), or

$$NF = \frac{N_{in} + N_{Dev-in}}{N_{in}} = \frac{1+1}{1} = 2$$

In dB, the noise factor of the amplifier in both situations is $NF_{dB} = 10 \cdot \log(NF) = 10 \cdot \log(2) = 3$ dB. To find the amplifier effective temperature use Eq. (4-10), $T_{eff} = (NF-1) \cdot T_0 = (NF-1) \cdot 290 = (2-1) \cdot 290 = 290$ K.

Example 4-3:

Find the noise factor and effective noise temperature of the attenuator for the conditions given (figure 4-3).

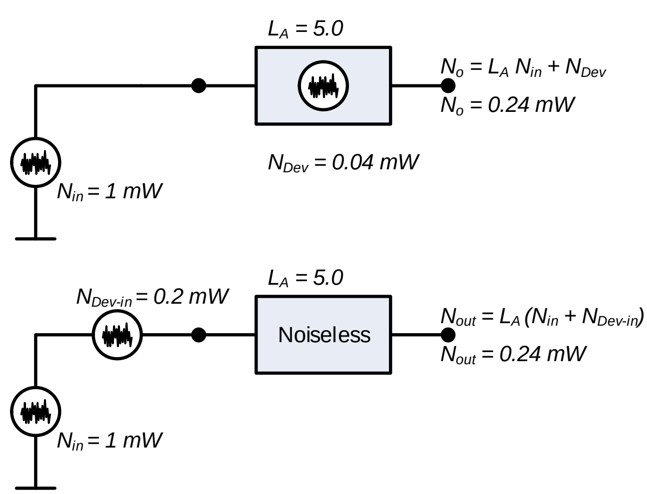


Fig. 4-3 – Noise factor of an attenuator. Two representations are shown as for the amplifiers in the previous example.

Solution: For either the upper or lower drawing

$NF = L_A = 5.0$

In dB, the noise factor of the attenuator from Eq. (4-14) is

$NF_{dB} = 10 \cdot \log(NF) = 10 \cdot \log(5) = 7 \text{ dB}$

To find the effective temperature use Eq. (4-10)

$T_{eff} = (NF - 1) \cdot 290 = (5-1) \cdot 290 = 1160 \text{ K}$

4-2. Noise factor of cascaded amplifiers

In Part III we solved for the equivalent input noise temperature of an amplifier cascade. In this section we will develop the equivalent (or composite) noise factor for an amplifier cascade. Consider two amplifiers with gains G_1 and G_2 and noise factors NF_1 and NF_2 (figure 4-4).

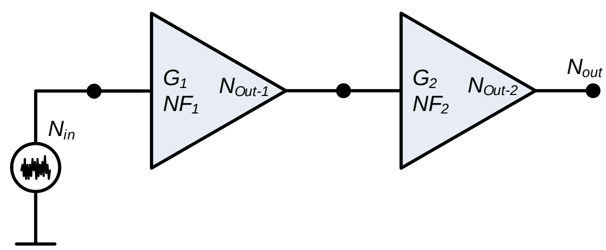


Fig. 4-4 ~ Two amplifiers in cascade. A noise source with noise power N_{in} is connected to the input of the first stage.

The output noise power of the cascade due only to the external source at the input is

$$N_{out-ext} = G_1 \cdot G_2 \cdot N_{in} \tag{4-16}$$

From Eq. (4-6) , the noise power at the output of the first stage due to its internal noise is

$$N_{Out-1} = G_1 \cdot (NF_1 - 1) \cdot N_{in} \tag{4-17}$$

The corresponding noise at the output of the second stage due to the first stage internal noise is

$$N_{Out-2-1} = G_2 \cdot N_{Out-1} = G_1 \cdot G_2 \cdot (NF_1 - 1) \cdot N_{in} \tag{4-18}$$

and the output noise of the second stage due to its own internal noise is

$$N_{Out-2} = G_2 \cdot \left(NF_2 - 1\right) \cdot N_{in} \tag{4-19}$$

The total noise at the output of the cascade is the sum of Eq. (4-16), (4-18) and (4-19), or

$$\begin{aligned} N_{Out} &= N_{out-ext} + N_{Out-2-1} + N_{Out-2} \\ N_{Out} &= G_1 \cdot G_2 \cdot N_{in} + G_2 \cdot \left(NF_2 - 1\right) \cdot N_{in} + G_1 \cdot G_2 \cdot \left(NF_1 - 1\right) \cdot N_{in} \end{aligned} \tag{4-20}$$

Substituting $G \cdot N_{Dev-in} = N_{Dev}$ into Eq. (4-4) and rearranging,

$$NF = \frac{G \cdot \left(N_{In} + N_{Dev-In}\right)}{G \cdot N_{In}} \tag{4-21}$$

The cascade can be considered a single device, and all internally generated noise referred to its input is $N_{Cascade-in} = N_{Dev-in}$. Also, $G_{Cascade} = G_1 \cdot G_2 = G$. Therefore,

$$NF_{Cascade} = \frac{G_{Cascade} \cdot \left(N_{in} + N_{Cascade-in}\right)}{G_{Cascade} \cdot N_{in}} \tag{4-22}$$

Since the external noise and internal noise referred to the input are increased by the cascade gain,

$$N_{Out} = G_{Cascade} \cdot \left(N_{In} + N_{Cascade-In}\right) \tag{4-23}$$

then Eq. (4-22) reduces to

$$NF_{Cascade} = \frac{N_{Out}}{G_{Cascade} \cdot N_{in}} = \frac{N_{Out}}{G_1 \cdot G_2 \cdot N_{in}} \tag{4-24}$$

Dividing both sides of Eq. (4-20) by $G_1 \cdot G_2 \cdot N_{in}$ and combining with Eq. (4-24)

$$NF_{Cascade} = 1 + \frac{(NF_2 - 1)}{G_1} + (NF_1 - 1) = NF_1 + \frac{(NF_2 - 1)}{G_1} \tag{4-25}$$

This concept can be extended to any number of stages. For m stages

$$NF_{Cascade} = NF_1 + \frac{(NF_2 - 1)}{G_1} + \frac{(NF_3 - 1)}{G_1 \cdot G_2} + \cdots + \frac{(NF_m - 1)}{G_1 \cdot G_2 \cdots G_{m-1}} \tag{4-26}$$

It is seen that the first stage has the most effect on the cascade noise factor. This result should not be surprising given the previous noise temperature analysis of an amplifier cascade. As a cross-check, we can derive the equivalent noise temperature of the cascade from Eq. (4-26) by combining it with Eq. (4-11) and letting $T_m = T_{eff}$, or

$$NF_m = 1 + \frac{T_m}{T_0} \text{ and } NF_{Cascade} = 1 + \frac{T_{Cascade}}{T_0} \tag{4-27}$$

Therefore,

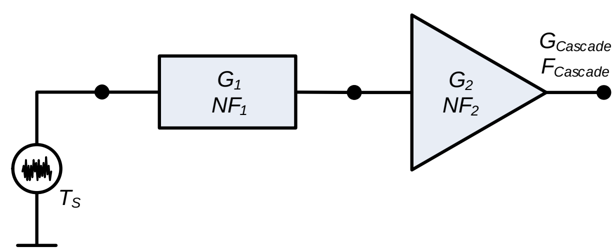
$$NF_{Cascade} = 1 + \frac{T_{Cascade}}{T_0} = 1 + \frac{T_1}{T_0} + \frac{\left(1 + \frac{T_2}{T_0}\right) - 1}{G_1} + \frac{\left(1 + \frac{T_3}{T_0}\right) - 1}{G_1 \cdot G_2} + \cdots + \frac{\left(1 + \frac{T_m}{T_0}\right) - 1}{G_1 \cdot G_2 \cdots G_{m-1}} \tag{4-28}$$

Cancelling terms

$$T_{Cascade} = T_1 + \frac{T_2}{G_1} + \frac{T_3}{G_1 \cdot G_2} + \cdots + \frac{T_m}{G_1 \cdot G_2 \cdots G_{m-1}} \tag{4-29}$$

which is identical to Eq. (2-40). As mentioned in Part II, we note that that $G_A = \frac{1}{L_A}$ (where G_A and L_A are both linear power ratios) so these concepts also apply to attenuation as shown in the following examples.

<u>Example 4-4</u>:
Find the noise factor of a cascade consisting of a lossy transmission line and an amplifier for the conditions shown (figure 4-5).



Parameter (dB)	Transmission Line	Amplifier
Gain or Loss	G_1 = 10 dB loss	G_2 = 20 dB gain
NF	NF_1 = 10 dB	NF_2 = 6 dB

Fig. 4-5 – Cascade consisting of an attenuator followed by an amplifier

<u>Solution</u>:
First convert the gains and noise factors to linear ratios

Parameter (ratio)	Transmission Line	Amplifier
Gain or Loss	$G_1 = 1/L_1 = 0.1$	$G_2 = 100$

NF	NF_1 = 10	NF_2 = 3.981

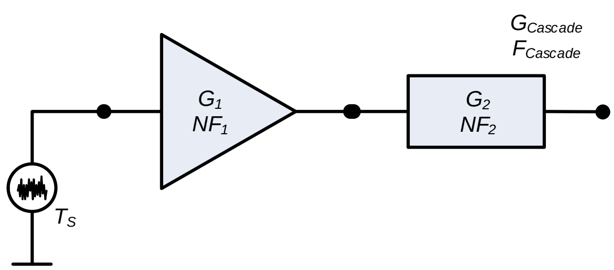
From equation (4-23),

$$NF_{Cascade} = NF_1 + \frac{(NF_2 - 1)}{G_1} = 10.0 + \frac{(3.981 - 1)}{0.1} = 39.81$$

$$NF_{Cascade,dB} = 10 \cdot \log(39.81) = 16 \text{ dB}$$

Comment: In this example, the composite noise factor is simply the sum of the transmission line loss and the noise factor of the amplifier, both in dB. Note that the noise factor does not depend on the amplifier gain. The composite noise factor could be improved dB for dB by using lower loss coaxial cable or by shortening it (moving the amplifier closer to the source). This explains why low noise amplifiers are located close to antennas (the source) and low loss connections and cable always are used in front of the low noise amplifier.

Example 4-5:
Find the noise factor of the cascade in the previous example except reverse the order of connection (figure 4-6).



Parameter (dB)	Amplifier	Transmission Line
Gain or Loss	G_1 = 20 dB gain	G_2 = 10 dB loss
NF	NF_1 = 6 dB	NF_2 = 10 dB

Fig. 4-6 – Cascade consisting of an amplifier followed by an attenuator

Solution:
First convert the gains and noise factors to linear ratios

Parameter (ratio)	Amplifier	Transmission Line
Gain or Loss	G_1 = 100	$G_2 = 1/L_2$ = 0.1
NF	NF_1 = 3.981	NF_2 = 10

From equation (4-25),

$$NF_{Cascade} = NF_1 + \frac{(NF_2 - 1)}{G_1} = 3.981 + \frac{(10-1)}{100} = 4.071$$

$$NF_{Cascade,dB} = 10 \cdot \log\left(4.071\right) = 6.1 \text{ dB}$$

Comment: The composite noise factor is dominated by the noise factor of the amplifier and the transmission line has only a small effect. If the amplifier had higher gain, say 30 dB or more, the transmission line noise factor would have negligible effect and the composite noise factor would be the same as the amplifier noise factor.

4-3. References

[Agilent 57-1] Fundamentals of RF and Microwave Noise Figure Measurement, Application Note 57-1, Document No. 5952-8255E, Agilent Technologies, Inc., 2010

[IEEE100] IEEE Std 100-1992, IEEE Standard Dictionary of Electrical and Electronics Terms, Institute of Electrical and Electronics Engineers, 1992

[ITU-R P-372.9] Recommendation ITU-R P-372.9, Radio Noise, International Telecommunications Union, Radio Communications Sector, 2007

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