Burial Depth of SAM-III Magnetometer Sensors

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1. Introduction

The output of the SAM-III magnetometer sensors (figure 1) varies with temperature. This variation can be controlled by thermally insulating the sensors. One of the best ways to insulate the sensors is to place them in a watertight fixture and bury the fixture outdoors in soil. The sensor fixture should be buried deep enough to at least eliminate sensor output variations caused by daily (diurnal) temperature changes. Seasonal temperature variations may be reduced by burying the sensor fixture deeper.

Figure 1 ~ SAM-III sensor. Three identical units are used in the 3-axis SAM-III geomagnetometer. They operate from 5 Vdc and have a pulse width modulated (PWM) output that is proportional to the magnetic flux density. Individual sensor length is approximately 61 mm. The sensors are temperature sensitive.

The sensors are connected to copper cables for power and signal. Because copper is a very good thermal conductor, these cables can affect the sensor temperature. Therefore, it is necessary to also bury the cables out to a reasonable distance from the sensors to insulate them. Installation details for the SAM-III sensors and cables can be found in the SAM-III Construction Manual [SAM3].

The next section briefly discusses heat flow in soil and is followed by sections that describe calculations to determine the required depth in typical soils to reduce the daily temperature variation to a given level. Additional calculations show that to eliminate seasonal or annual variations, relatively deep burial is required but more moderate depths can achieve acceptable performance. These calculations provide first-order results adequate for all but the most demanding applications.

2. Heat flow in soil

The temperature at any soil depth is affected by the temperature gradient between it and the atmosphere in contact with the soil surface. Soil is thermally conductive and heat flows in it from higher to lower temperatures. Generally, during the day the surface soil is warmed by the Sun and at night it is cooled, so heat flows into the soil during the day and out during the night resulting in cyclic temperature variations at and near the surface.

The type of soil and its porosity and moisture content affect heat flow. Soil heat flow characteristics are different and highly variable for unfrozen and frozen soils. Porous soils drain relatively quickly after rain, so their heat-flow characteristics will be transient during rainy periods. Soils that hold moisture for long periods will have different thermal characteristics than dry soils. Loose snow cover and foliage tend to retard heat flow into and out of the soil surface. Many soils are stratified (for example, a layer of peat over sandy-gravel and then bedrock), which further complicates their heat flow characteristics.

The atmospheric conditions at the soil surface have a natural periodicity, which controls the energy input into the soil and thus the soil temperature. A daily (diurnal) temperature cycle is superposed on a seasonal cycle. These cycles usually are modeled as pure harmonic (sinusoidal) functions of time about some mean value of temperature. However, the actual cycles are perturbed by irregular weather systems in the area such as wind, cloudiness and rain.

The available heat energy on the surface decreases with depth and takes time to travel into and out of the soil. Therefore, the amplitude of temperature variations at any depth is damped and also delayed with respect to the surface. The time delay becomes more pronounced with increasing depth. At some relatively large depth the variations are damped out and the soil temperature is constant (figure 2).

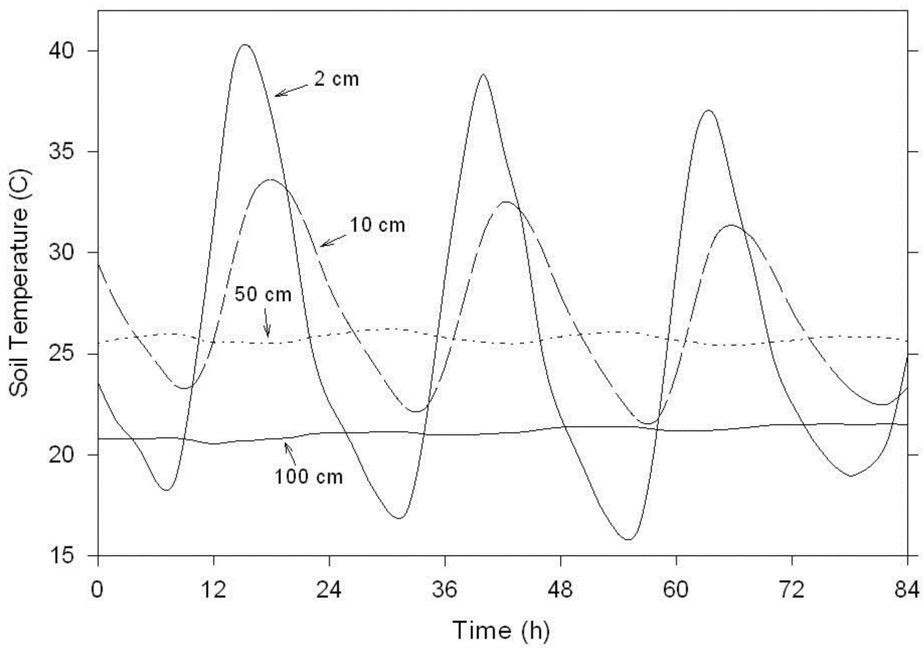

Figure 2 ~ Temperature variations over a 3.5 d period at various depths in cm. The amplitude of soil temperature variations at 10 cm depth is about one-half the amplitude at 2 cm. At 50 cm depth, the daily temperature variations are very small and at 100 cm almost completely damped out. There is a few hours delay between the maximum (or minimum) temperatures at 2 cm and 10 cm depths and about 12 h delay between 2 cm and 50 cm depths.

Image from [Hillel].

Heat flow in any substance is theoretically described by differential equations. However, most soils defy complete theoretical analysis and on-site measurements are used to quantify their heat flow characteristics. At the expense of accuracy, simplifying assumptions and “typical” or “average” characteristics can be used rather than measurements.

3. Diurnal Calculations

This section develops an approximate method to calculate the depth required to reduce (damp) the effects of daily temperature variations. The concepts will be extended to annual and seasonal variations in the next section. The simplest representation of the natural thermal variations is to assume that (1) at all depths the soil is homogenous (of the same type and characteristics) and (2) its temperature oscillates as a sinusoidal function of time around an average value that is the same for all depths. If the soil surface is initially (time t = 0) at the average temperature $\overline{T}$, the surface temperature can be expressed by

$$T(0,t) = \bar{T} + A_0 \cdot \sin(\omega \cdot t) \quad (1)$$

where

$T(0,t)$ Temperature at the soil surface as a function of time t (°C)

$\bar{T}$ Average temperature (°C)

A_0 Temperature amplitude range from maximum (or minimum) to the average temperature (°C)

ω Radian frequency of the periodic variation (rad/time unit). $\omega = 2 \cdot \pi / P$ for period *P*. Typical time units are seconds, hours and days.

Equation (1) can be plotted to show the surface temperature variation for a 24 h period (figure 3).

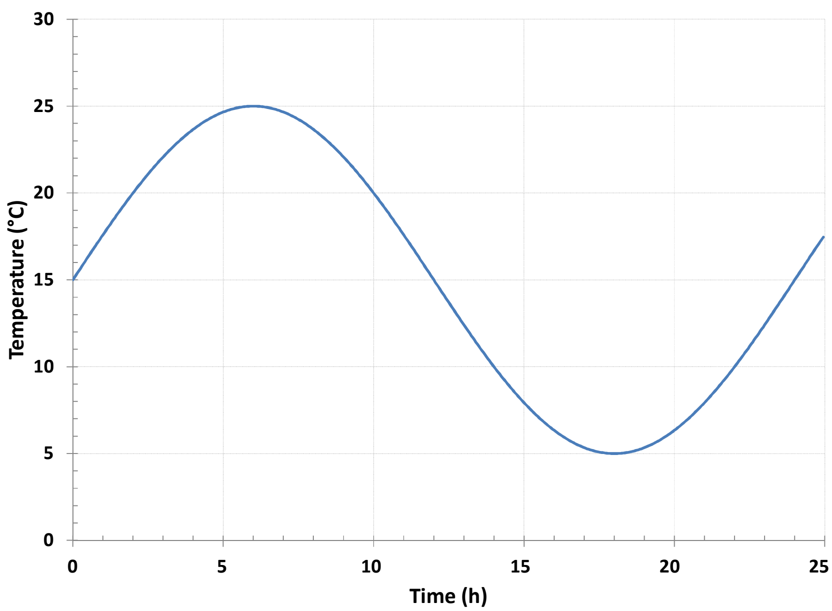

Figure 3 ~ Simple sinusoidal variation of the daily temperature at the soil surface for an average surface temperature of 15 °C and amplitude range of 10 °C.

Assuming the same sinusoidal variation and average temperature, the temperature at any depth is

$$T(z,t) = \bar{T} + A_z \cdot \sin\left[\omega \cdot t + \varphi(z)\right] \quad (2)$$

where

$T(z,t)$ Temperature of the soil as a function of time t at depth z (°C)

z Depth (m)

$\bar{T}$ Average temperature (°C)

A_z Temperature amplitude range at depth z (°C)

ω Radian frequency of the periodic variation (rad/time unit). $\omega = 2 \cdot \pi / P$ for period *P*. Typical time units are seconds, hours and days.

$\varphi(z)$ Phase delay at depth z (rad)

Both A_z and $\varphi(z)$ are functions of depth z but not time t. A solution to the differential equations that describe heat flow shows that for a damping depth, *d*

$$T(z,t)=\overline{T}+A_0\cdot e^{-z/d}\cdot\sin\left[\omega\cdot t-z/d\right] \tag{3}$$

where

d Damping depth (m) at which the temperature amplitude variation decreases to the fraction 1/e (= 0.368) of the surface amplitude A_D. For example, if A_0 at the surface is 10 °C, then at the damping depth the amplitude A_z would be 3.68 °C.

The damping depth is related to the soil properties including type of soil, moisture content and porosity. It can be defined by

$$d=\left(2\cdot D_H/\omega\right)^{1/2} \tag{4}$$

$$d=\left(2\cdot P\cdot D_H/2\cdot\pi\right)^{1/2} \tag{4a}$$

where D_H is the thermal diffusivity of the soil. The thermal diffusivity is a measure of the soil thermal inertia, indicating how rapidly heat can move through the soil (the thermal diffusivity is defined mathematically below). As seen in equation (4a), the damping depth is proportional to the square root of the period of the temperature fluctuations. Therefore, the damping depth for the annual variation is $\sqrt{365}\approx 19$ times larger than for the diurnal variation in the same soil.

Equation (3) is plotted to show the temperature variation at various depths over a 24 h period (figure 4). Equation (3) can be modified to include a time shift so that the peak temperature occurs at an arbitrary time of day. This typically is used to place the peak surface temperature in the mid-afternoon around 2 PM local time, although considerably different shifts are needed at higher latitudes depending on the season. For the general case

$$T(z,t)=\overline{T}_D+A_D\cdot e^{(-z/d_D)}\cdot\sin\left[\omega_D\cdot t+\varphi_0-z/d_D\right] \tag{5}$$

$$A_D=\left(T_{\max}-T_{\min}\right)/2 \tag{6}$$

$$\omega_D=2\cdot\pi/P_D \tag{7}$$

$$\varphi_0=-\omega_D\cdot t_0 \tag{8}$$

$$d_D=\sqrt{P_D\cdot D_H/\pi} \tag{9}$$

$$D_H=\lambda/c_V \tag{10}$$

where

$\overline{T}_D$ Daily average temperature at the soil surface (°C). To calculate worst-case, this typically would be for the day of the year with the highest amplitude

A_D	One-half the peak amplitude of daily temperature variation (°C)
z	Depth (m); $0 \le z \le \infty$
d_D	Diurnal damping depth (m)
ω_D	Radian frequency of the daily variations (rad h^{-1})
P_D	Period of daily temperature variation (h); P_D = 24 h
φ_0	Phase shift (offset) for arbitrary zero point (rad); the phase constant is –8.3 rad to obtain maximum soil surface temperature in the afternoon at 2 PM local time
t	Time (h)
t_0	Time shift for arbitrary zero point (h)
D_H	Thermal diffusivity (m^2 s^{-1}) in terms of soil thermal conductivity λ and volumetric heat capacity c_V
λ	Thermal conductivity (W m^{-1} K^{-1})
c_V	Volumetric heat capacity (J m^{-3} K^{-1})

Note: The thermal conductivity λ and volumetric heat capacity c_V may be given in some literature with celsius temperature units (°C); these values are interchangeable with values given in kelvin (K) for the calculations discussed here.

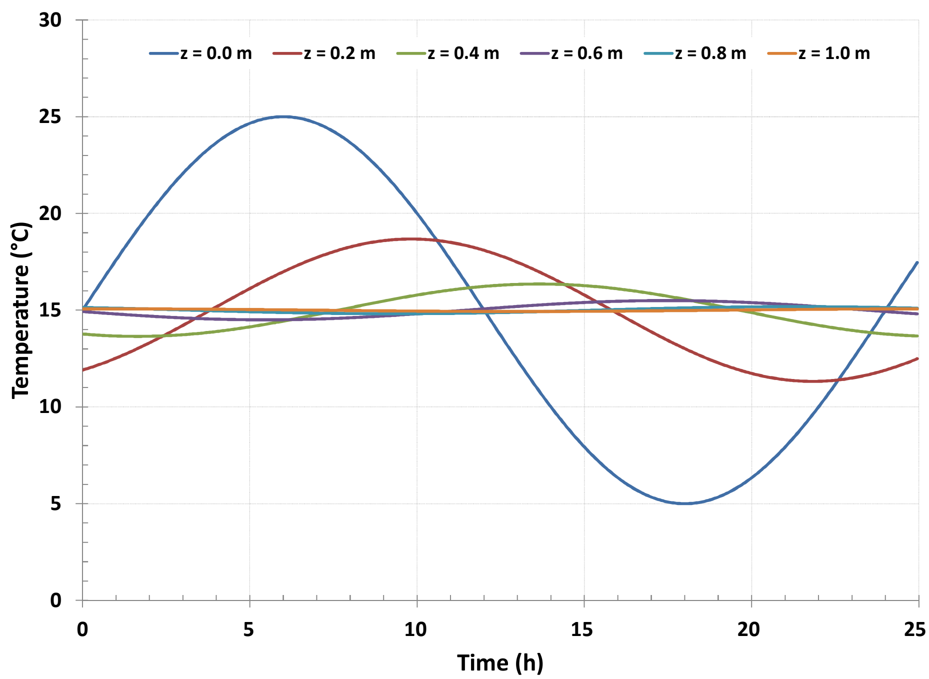

Figure 4 ~ Simple sinusoidal representation of the temperature at the soil surface (z = 0) and various other depths to 1.0 m over a 24 h period for an average surface temperature of 15 °C and amplitude range of 10 °C. Starting time is arbitrary, and the damping depth is 0.2 m. Note that the amplitude decreases and delay increases with depth.

Soil characteristics: The maximum, minimum and average daily temperatures for a given location can be determined from national weather services. The thermal characteristics of typical soils are given below (table 1). The thermal conductivity and volumetric heat capacity may be used to calculate the thermal diffusivity using equation (10) and that may be used to calculate the damping depth from equation (9).

Table 1 ~ Typical soil thermal characteristics (SI units upper table; CGS units lower table). Data source: [Hillel] with unit conversion in upper table

Soil type	Thermal conductivity, λ W m^{-1} K^{-1}	Volumetric heat capacity, c_V 10^6 J m^{-3} K^{-1}	Diurnal damping depth, d_D m
Sand	0.3 to 2.2	1.3 to 2.9	0.08 to 0.14
Clay	0.25 to 1.6	1.3 to 2.9	0.07 to 0.12
Peat	0.06 to 0.5	1.5 to 4.8	0.03 to 0.05
Snow	0.06 to 0.7	0.2 to 2.1	0.09 to 0.10

Soil type	Thermal conductivity 10^{-3} cal cm^{-1} s^{-1} °C^{-1}	Volumetric heat capacity cal cm^{-3} °C^{-1}	Diurnal damping depth, d_D cm
Sand	0.7 to 5.2	0.3 to 0.7	8.0 to 14.3
Clay	0.6 to 3.8	0.3 to 0.7	7.4 to 12.2
Peat	0.14 to 1.2	0.35 to 1.15	3.3 to 5.4
Snow	0.15 to 1.7	0.05 to 0.5	9.1 to 9.7

A range of damping depths is given in the table for each soil type. Generally, the lower values apply to dryer soils and the higher values to wetter soils. Note that there is not a great deal of difference in the range of damping depths for sand and clay. Soil types such as sandy-gravel and sandy-clay are very common. Also note that peat, which is mostly decomposed organic matter, has the lowest damping depth and, thus, is a relatively good thermal insulator, particularly if it is dry.

Sensor thermal characteristics: A limited number of measurements indicate the temperature coefficient of the SAM-III sensors is on the order of –100 to –150 nT °C^{-1}. These values are not statistically significant but they may be used as a starting point. Greater accuracy can be achieved by measuring the temperature coefficient of each sensor or the average value of a large number of sensors.

Depth calculation: It is necessary to determine the multiplication or damping factor that will reduce the temperature amplitude variation at depth z to the allowable range. Equation (3) indicates that the amplitude of temperature variation is damped by the factor $e^{-z/d}$. Let ΔT be the amplitude damping factor. Then

$$\Delta T = e^{(-z/d_D)} \tag{11}$$

Solving for the ratio z/d_D gives

$$z/d_D = -\ln(\Delta T) \tag{12}$$

and solving equation (12) for the depth z gives

$$z = -d_D \cdot \ln(\Delta T) \tag{13}$$

Example 1: To reduce the sensor output variation due to daily temperature changes to the equivalent of 1 nT and assuming a sensor temperature coefficient of –100 nT °C^{-1}, it is necessary to reduce the temperature changes at the sensor to 1 nT/100 nT °C^{-1} = 0.01 °C, or a factor of 0.01. If the damping depth d_D of the soil is 0.1 m (sandy clay, clay or sand), then from equation (13)

$$z = -d_D \cdot \ln(\Delta T) = -0.1 \cdot \ln(0.01) = 0.46 \text{ m}$$

This depth corresponds to an amplitude damping factor equal to e$^{-4.61}$, corresponding to the ratio $z/d_D = 4.61$.

Example 2: Assuming a coefficient of -150 nT °C^{-1} and to reduce the sensor output changes to 3 nT in the same soil, the temperature variations would have to be reduced to 3 nT/150 nT °C^{-1} = 0.02 °C. From equation (13)

$$z = -d_D \cdot \ln(\Delta T) = -0.1 \cdot \ln(0.02) = 0.39 \text{ m}$$

This depth corresponds to an amplitude damping factor equal to e$^{-3.91}$, or the ratio $z/d_D = 3.91$.

A design margin should be applied to all depth calculations to account for uncertainties in the soil characteristics and sensor temperature coefficient. For example, a design margin of 1.5 or 2.0 may be appropriate, in which case the burial depths would be 1.0 and 0.8 m, respectively, for the above examples.

4. Annual Calculations

Annual temperature variations include the effects of changing seasons with a temperature maximum in the summer and minimum in the winter (figure 5). Procedures similar to those described in the previous section are used to determine the depth required to reduce temperature fluctuations over an annual temperature cycle. In this case, the equation that describes the temperature as a function of time and depth has both daily (subscript D) and annual (subscript A) components as seen in equation (14).

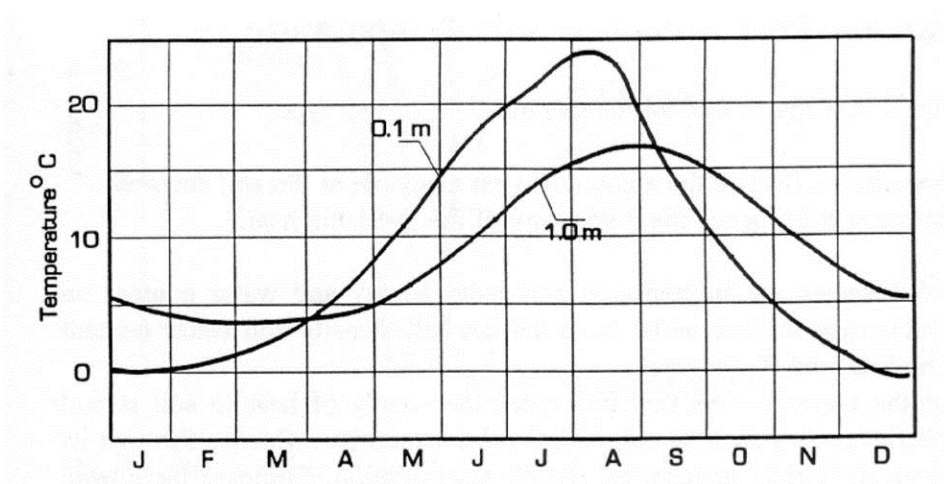

Figure 5 ~ Annual temperature variations over a 1 yr period at 0.1 and 1.0 m depths. Daily variations throughout the seasons are not shown. Note that the delay in the minimum soil temperature during winter months is much longer than summer, indicating that heat flow in cold soils during winter is slower than in warm and possibly wetter soils during summer.

Image from [Spring].

$$T_A(z,t) = A_D \cdot e^{(-z/d_D)} \cdot \sin[\omega_D \cdot t - \varphi_D - z/d_D] + \overline{T}_A + A_A \cdot e^{(-z/d_A)} \cdot \sin[\omega_A \cdot t - \varphi_A - z/d_A] \tag{14}$$

$$d_A = \sqrt{P_A \cdot D_H / \pi} \tag{15}$$

$$\omega_A = 2 \cdot \pi / P_A \tag{16}$$

$\overline{T}_A$ Annual average temperature at the soil surface (°C)

A_A One-half the peak amplitude of annual temperature variation (°C)

P_A Period of annual temperature variation (h); P_A = 8 760 h ($31.5 \cdot 10^6$ s).

d_A Annual damping depth (m)

ω_A Radian frequency of the annual variations (rad h^{-1} or rad s^{-1})

φ_A Phase shift (offset) for arbitrary zero point (rad); the phase constant is –8.3 rad to obtain maximum soil surface temperature during July (northern hemisphere).

Note: All time units must be compatible (for example, all days, hours or seconds and not a mixture)

Only the annual components of equation (14) are defined above; daily components were defined previously. The annual damping depth (table 2) is based on the diurnal damping depths in table 1 increased by the factor $\sqrt{365} = 19$; that is, $d_A = 19 \cdot d_D$. If equation (15) is used to calculate the annual damping depth from the table values, the period P_A must be in seconds. Thermal diffusivity D_H may be obtained from equation (10).

Table 2 ~ Typical soil thermal characteristics (SI units upper table; CGS units lower table). Source: Table 1 with calculation of the annual damping depth from the diurnal damping depth as indicated in the text.

Soil type	Thermal conductivity, λ W m^{-1} K^{-1}	Volumetric heat capacity, c_V 10^6 J m^{-3} K^{-1}	Annual damping depth, d_A m
Sand	0.3 to 2.2	1.3 to 2.9	1.5 to 2.7
Clay	0.25 to 1.6	1.3 to 2.9	1.4 to 2.3
Peat	0.06 to 0.5	1.5 to 4.8	0.6 to 1.0
Snow	0.06 to 0.7	0.2 to 2.1	1.7 to 1.8

Soil type	Thermal conductivity 10^{-3} cal cm^{-1} s^{-1} $°C^{-1}$	Volumetric heat capacity cal cm^{-3} $°C^{-1}$	Annual damping depth, d_A cm
Sand	0.7 to 5.2	0.3 to 0.7	152 to 272
Clay	0.6 to 3.8	0.3 to 0.7	141 to 232
Peat	0.14 to 1.2	0.35 to 1.15	63 to 103
Snow	0.15 to 1.7	0.05 to 0.5	173 to 184

The daily temperature variations ride on the annual variations (figure 6). Generally, the diurnal variations can be ignored when making annual calculations for depth.

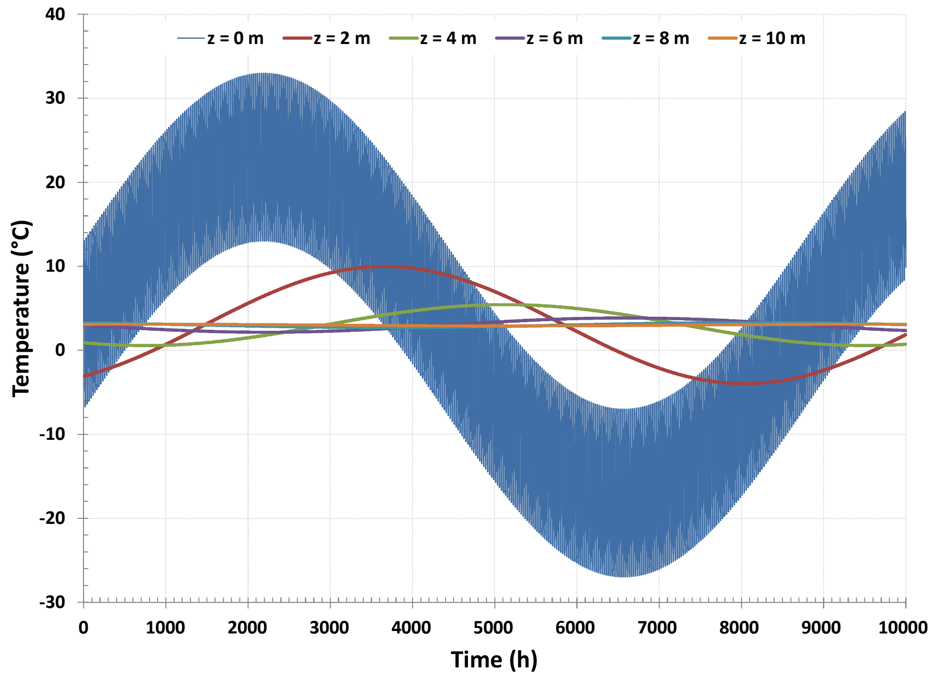

Figure 6 ~ Simple sinusoidal representation of the temperature at the soil surface (z = 0) and various other depths to 1.0 m over 1 yr (8 760 h) period for an average annual surface temperature of 3 °C and annual amplitude range of 20 °C. Starting time is arbitrary. The blue trace is the surface temperature and shows the daily variations superposed on the annual variation. The daily variations are insignificant below about 3x the diurnal damping depth. In this example, the annual damping depth is 1.9 m and the diurnal damping depth is 0.1 m. This plot assumes the daily temperature variations are the same throughout the year.

The following examples use the same data as for the diurnal variations in the previous section.

Example 3: To reduce the output variation due to temperature variations over a year to the equivalent of 1 nT and assuming a sensor temperature coefficient of –100 nT °C^{-1}, it is necessary to reduce the temperature changes at the sensor to 1 nT/100 nT °C^{-1} = 0.01 °C (as before). If the annual damping depth d_A of the soil is 1.9 m

$$z = -d_A \cdot \ln(\Delta T) = -1.9 \cdot \ln(0.01) = 8.7 \text{ m}$$

Example 4: Assuming a coefficient of –150 nT °C^{-1} and to reduce the sensor output changes to 3 nT in the same soil, the temperature variations would have to be reduced to 3 nT/150 nT °C^{-1} = 0.02 °C, and

$$z = -d_D \cdot \ln(\Delta T) = -1.9 \cdot \ln(0.02) = 7.4 \text{ m}$$

As with daily variations, a design margin would be applied in the two examples above leading to greater depths. Even without a design margin such depths would require expensive construction, not only in placing the sensor fixture but also in the sensor fixture itself. Therefore, it is worth considering less stringent temperature variations.

The annual temperature amplitude damping factor is similar to equation (11), or

$$\Delta T = e^{(-z/d_A)} \tag{17}$$

Equation (17) can be plotted for various annual damping depths d_A (figure 7).

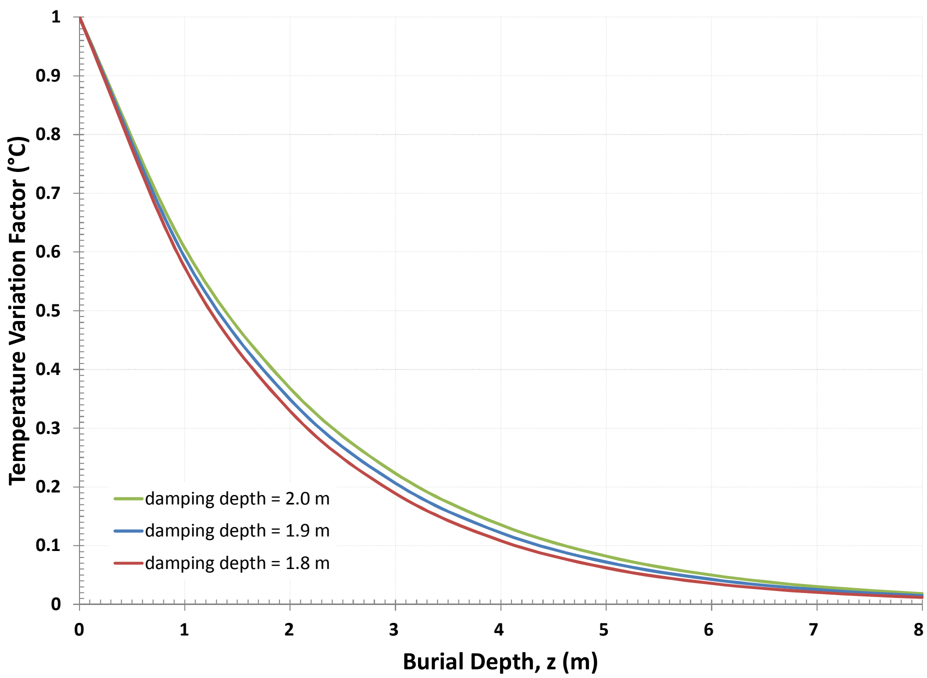

Figure 7 ~ Temperature variation factor as a function of depth for three annual damping depths.

Example 5: Using equation (17) for burial depth z = 3 m and damping depth d_A =1.9 m (as above),

$$e^{(-z/d_A)} = e^{-3/1.9} = 0.206\ °\mathrm{C}$$

If the sensor temperature coefficient is –150 nT $°C^{-1}$, the sensor output variation throughout the year will be $0.206 \cdot 150 = 30.9$ nT, which probably is tolerable in most installations. From the plot, decreasing the depth to 2 m increases the factor to about 0.35. For the same coefficient, the sensor output would change by $0.35 \cdot 150 = 52.5$ nT.

As with diurnal calculations, the above should be used with a design margin. Also, it should be remembered that soil stratification is more likely at greater depths and may play a significant role in the temperature damping.

5. Conclusions

As a first order approximation it is possible to reduce the SAM-III sensor output variations due to daily temperatures by burying the sensors at least 0.5 m below the surface (includes no design margin). Complete elimination of annual variations requires burial at impractical depths but a depth of 2 or 3 m provides a significant reduction. Of course, these conclusions depend on soil thermal characteristics, which have been generalized in this paper.

6. References

[Hillel] Hillel, D., Introduction to Soil Physics, Academic Press, 1982

[SAM3] SAM-III Simple Aurora Monitor, 3-Axis Geomagnetometer ~ Construction Manual, available here: http://www.reeve.com/Documents/SAM/SAM3ConstructionManual.pdf

[Spring] http://www.pci.tu-bs.de/agbauerecker/VorlesungSolarthermieUndWaermepumpen2014/WaermeeintragErdeLecture17_Spring2004.pdf

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